Introduction to Mathematical Proof

Course Description

Dr. Patrick Rault

Math 239, Spring 2012

MWF, 12:30-1:20pm

Basic Course Information

- Description: The course will provide an introduction to the language of advanced mathematics and to mathematical proof. It will emphasize rigorous argument and the practice of proof in various mathematical contexts. Topics will include logic, set theory, cardinality, methods of proof, and induction. Other mathematical topics chosen at the discretion of the instructor will be included as material through which proving skills will be honed.

Prerequisites: Math 222 (Calculus 2)

This course is an excellent preparation for: Every 300-level mathematics class

Inquiry-Based Learning:
This course involves a high degree of inquiry-based active-learning, which emphasizes **doing mathematics** over learning content. This focus will allow you to share your ideas with the class; it will be a fun, relaxed, and enjoyable environment. Lecturing will be held to a minimum.

The key difference from traditional offerings will be as follows. Students will anticipate each class by preparing basic materials. The in-class time will be spent in discussion and bringing nuance to the prepared material. We can then RETHINK the issues—concepts and proofs—to hone these ideas and receive feedback from classmates and instructors in class.

A recent research study[1] about the growing nationwide trend toward Inquiry Based Learning (IBL) courses states that it "benefited students in multiple, profound, and perhaps lasting ways. Learning gains and attitudinal changes were especially positive for groups that are often under-served by traditional lecture-based approaches, including women and lower-achieving students […] Yet there was no evidence of negative consequences of IBL for men, high-achieving students, older and more experienced students: these groups too made gains greater than their non-IBL peers."

[1] Assessment & Evaluation Center for Inquiry-Based Learning in Mathematics (2011). *Evaluation of the IBL Mathematics Project: Student and Instructor Outcomes of Inquiry-Based Learning in College Mathematics*. (Report to the Educational Advancement Foundation and the IBL Mathematics Centers) Boulder, CO: University of Colorado, Ethnography & Evaluation Research.

Who would benefit from this course?

1. Those who want or need practice speaking about mathematics to an audience of their peers, e.g. prospective teachers or professors, or those planning to pursue a career which includes public speaking.
2. Those who have difficulty with proofs and want to go back to the basics to improve their fundamental proving skills.
3. Those who are comfortable with their proving skills and want to step somewhat beyond the usual assignments by creating their own theorems and discovering proofs for the main theorems of the class.
4. Those who wish to experience mathematics and get a small taste of what mathematics research is: make and prove conjectures, write your own theorems and lemmas, and think creatively outside the box.
5. Non-majors who want a taste of upper level mathematics in one elective.

Grading:
Written homework, student contributions in class (based on quality, not quantity), and exams will all be graded in this class.

Alternatives:
Several other sections of this class are taught every semester. Each instructor brings a unique experience to the classroom, with different aims and goals. You are encouraged to discuss these styles with your peers and instructors to determine which fits you at this current time in your life. For example, you may desire more practice presenting in front of others because of your chosen career path, or you may wish to take a class which involves learning how to use LaTeX to typeset mathematics.

Other Inquiry Based Learning (IBL) courses are also offered by various professors in the mathematics department. Dr. Nicodemi and Dr. Rault will be co-teaching Abstract Algebra (Math 330) as a 4-credit class in an IBL style this Spring.

Learning objectives

- Understand and apply the logical structure of proofs and work symbolically with connectives and quantifiers to produce logically valid, correct and clear arguments.
- Perform set operations on finite and infinite collections of sets and be familiar with properties of set operations.
- Determine equivalence relations on sets and equivalence classes
- Work with functions and in particular bijections, direct and inverse images and inverse functions
- Construct direct and indirect proofs and proof by induction and determine the appropriateness of each type in a particular setting. Analyze and critique proofs with respect to logic and correctness.
- Successfully work at problem solving via practice in unraveling abstract definitions, creating intuition-forming examples or counterexamples, and prove conjectures. Broadly speaking, students are expected to gain an understanding of what mathematics research is.
- Skill in giving and critiquing presentations. Increased comfort in discussing mathematics.
- Written communication of the solution of a mathematical problem via proof, with each step following from previous steps. Students will understand and produce rigorous arguments (proofs).

NOTE: The low amount of mathematical content on this list is intentional. We will focus on the process of doing mathematics as opposed to covering a high amount of content.