TOP HAT

The "join code" is 08724
To participate in the cell phone polling,
Text to (315) 636 -0905 the four digit code for each question followed by the response.

Which of the following best describes you?
a.) math teacher
b.) physics teacher
c.) science teacher (not-physics)
d.) technology teacher
e.) other

Discovering the Subatomic World

Dr. Kurt Fletcher
SUNY Distinguished Teaching Professor
SUNY Geneseo

NYS Master Teacher Program
March 2016

Minicourse Outline

1.) The electron

2.) The atomic nucleus

3.) and Beyond!

Discovering the Subatomic World 3

Our Approach

- How do we know what we know?
- More experimental than theoretical
- Somewhat historical
- Engage in active learning!

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Things we know about the atomic nucleus

1.) The nucleus is small

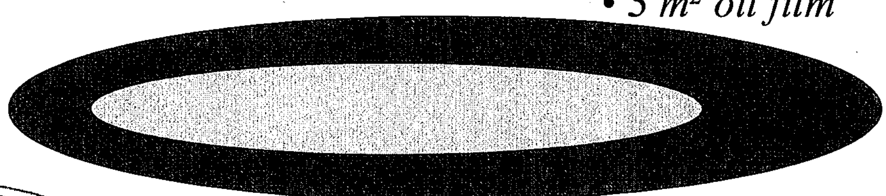
2.) The nucleus is massive

3.) the nucleus is positively charged

4.) The nucleus contains protons and neutrons

Victor Weisskopf – Oil drops on water

- *1 mm^3 oil drop*
- *3 m^2 oil film*



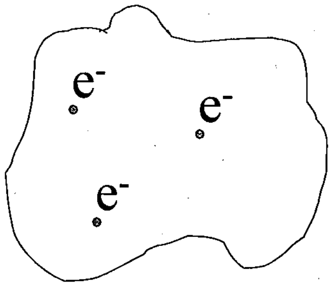
t=Volume/area = 0.3 nm = 3×10^{-10} m

- *thickness of an atom ≤ 0.3 nm*

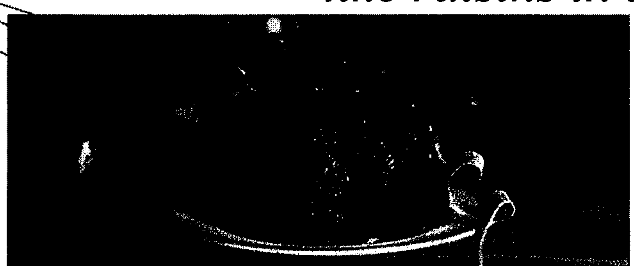
Victor Weisskopf, Knowledge and Wonder, 1979

Models of Atoms:

a.) Thompson's "Plum Pudding" model



- *Positively charged "blob" of matter. (pudding)*
- *e^- sprinkled throughout, like raisins in a*



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Ernest Rutherford –

"Can we examine the positive charge?"

- *Radioactive elements (Radon) emit positively charged alpha particles (^4He ions)*
- *Positive charges repel other positive charges*
- *Gold foil – malleable -1 μm or 10,000 atoms thick*

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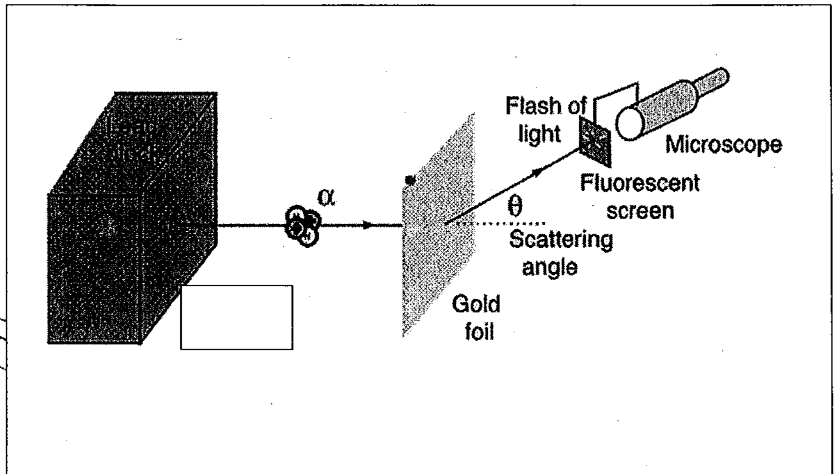
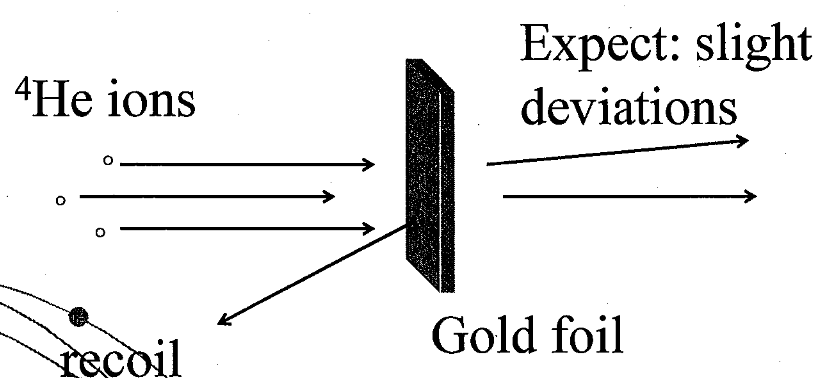


http://hyperphysics.phy-astr.gsu.edu/hbase/rutsca.html

Geiger and Marsden's Experiment (1910)

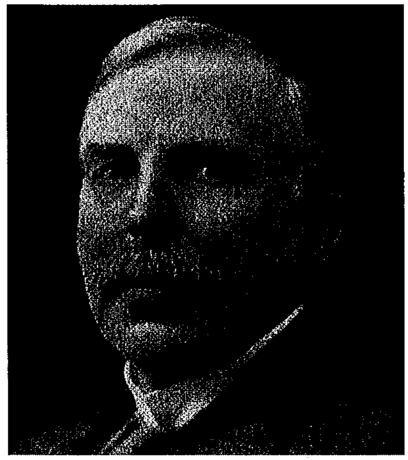


Expectation: chance of recoil is $\sim 10^{-3000}$

Experiment: chance of recoil is $\sim 10^{-4}$

Ernest Rutherford (G & M's advisor):

"It was quite the most incredible event that ever happened to me in my life. If was as incredible as if you fired a 15" shell at a piece of tissue paper and it came back and hit you."



http://kids.britannica.com/elementary/art-195677/Ernest-Rutherford-was-a-central-figure-in-the-study-of

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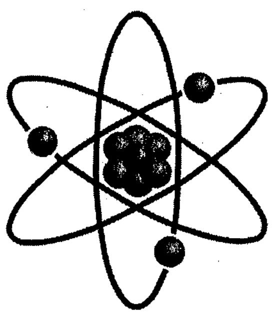

https://en.wikipedia.org/wiki/File:Stylised_Lithium_Atom.svg

- *Most of atom's mass is concentrated in a dense, small "nucleus"*

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Z_1 e(Z_2 e)}{r^2}$$

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http://phet.colorado.edu/en/simulation/rutherford-scattering

Rutherford Scattering Formula

$$N(\theta)=\frac{N_i n t}{16}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{Z_1^{\,2}Z_2^{\,2}}{r^2(K)^2\sin^4\left(\frac{\theta}{2}\right)}$$

Number of projectiles scattered per unit area into ring at θ

Here "r" is the target-to-detector distance

TOP HAT

$$N(\theta) = \frac{N_i n t}{16}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{Z_1^{\,2} Z_2^{\,2}}{r^2 (K)^2 \sin^4\left(\frac{\theta}{2}\right)}$$

For a particular Rutherford scattering experiment, 3144 alpha particles scatter into a detector placed at 30°. How many are scattered into an identical detector placed at 160° ?

Ans: 15

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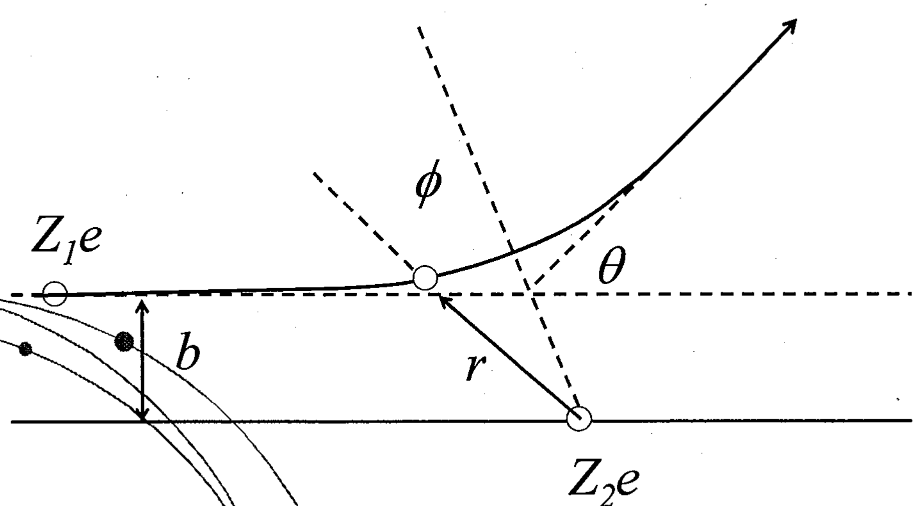
Derivation/Digression



Images inspired by Thornton and Rex, Modern Physics, 4th ed. Brooks Cole

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$$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$

$$d\vec{p} = \int \vec{F}\, dt$$

$$\Delta\vec{p} = \int \vec{F}_{\Delta\vec{p}}\, dt$$

FORCE ALONG $\Delta\vec{p}$

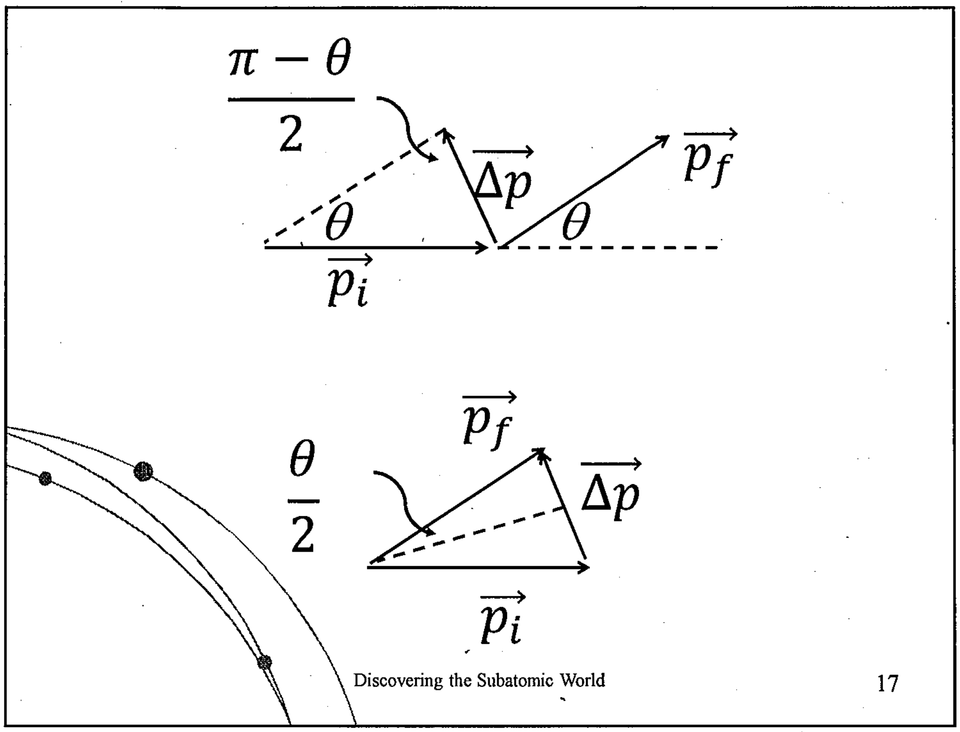
$$\Delta\vec{p} = \vec{P}_f - \vec{P}_i$$

$$\Delta p = 2(m v_0) \sin \theta/2$$

$$2 m v_0 \sin \theta/2 = \int F_{\Delta p}\, dt$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{Z_1 Z_2 e^2}{r^2} \hat{r}$$

$$F_{\Delta p} = F \cos\phi$$


$\frac{\pi - \theta}{2}$
$\overrightarrow{\Delta p}$
$\overrightarrow{p_f}$
θ
θ
$\overrightarrow{p_i}$
$\overrightarrow{p_f}$
$\frac{\theta}{2}$
$\overrightarrow{\Delta p}$
$\overrightarrow{p_i}$
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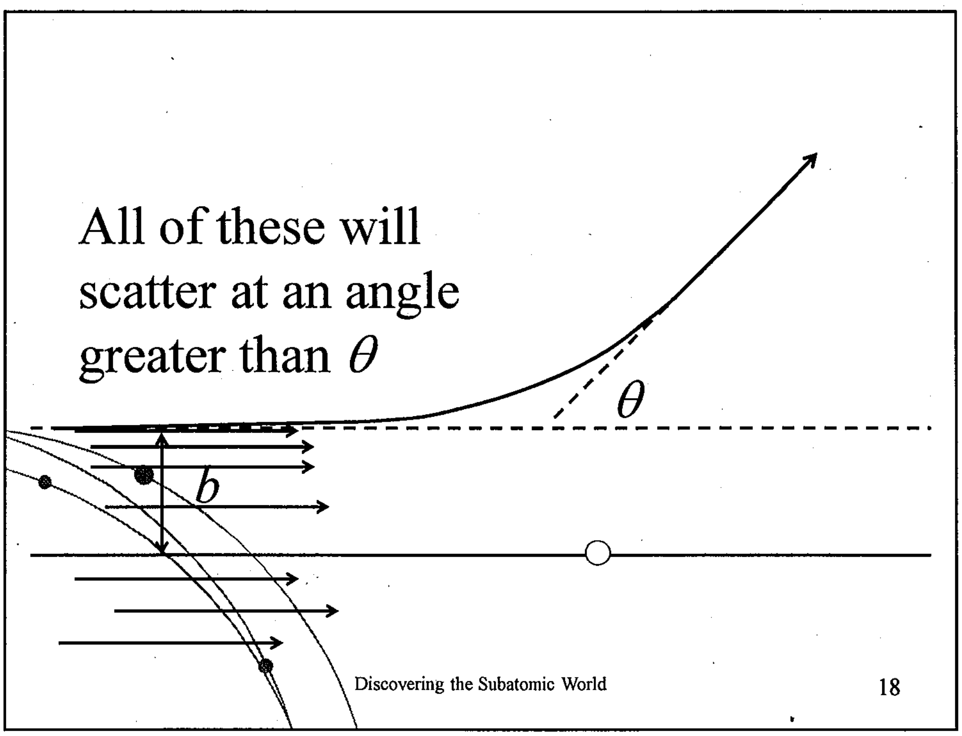
All of these will scatter at an angle greater than θ
θ
b
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$$2 m v_0 \sin\theta/2 = \int \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r^2} \cos\phi \, dt$$

ℓ IS CONSERVED "$I\omega$"

$$m v_0 b = m r^2 \frac{d\phi}{dt}$$

$$r^2 = \frac{v_0 b}{d\phi/dt}$$

$$2 m v_0 \sin\theta/2 = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0} \int \frac{\cos\phi}{v_0 b} \frac{d\phi}{dt} dt$$

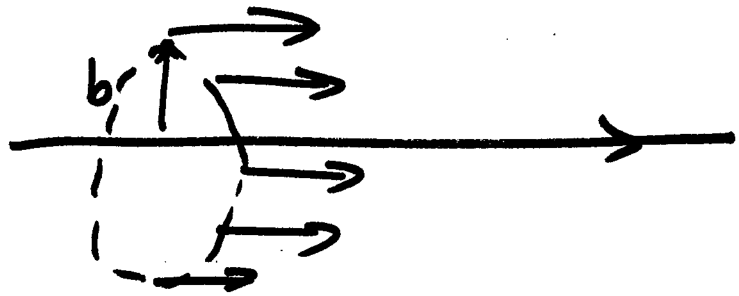
$$= \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0} \frac{1}{v_0 b} \int_{-\frac{(\pi-\theta)}{2}}^{\pi-\theta/2} \cos\phi \, d\phi$$

$$\frac{8\pi\epsilon_0 m v_0^2}{Z_1 Z_2 e^2} \sin\theta/2 = 2\cos(\theta/2)$$

$$b = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 m v_0^2} \cot\theta/2$$

$$b = \frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K} \cot\theta/2$$

CANNOT "SET" b



CALL "CROSS SECTION"

$$\sigma = \pi b^2$$

ALL PROJECTILES WITHIN THIS CIRCLE WILL SCATTER MORE THAN θ

$$\# \frac{\text{TARGETS}}{\text{AREA}} = n\,t$$

Volume density (n)

THICKNESS (t)

TARGETS = ntA

PROBABILITY OF SCATTERING MORE THAN θ = $\dfrac{\text{AREA of TARGETS}}{\text{TOTAL AREA}}$

$$f = \frac{ntA\,\sigma}{A} = nt\pi b^2$$

$$f = nt\pi \left(\frac{z_1 z_2 e^2}{8\pi\epsilon_0 K}\right)^2 \cot^2 \theta/2$$

$$\frac{df}{d\theta} = -nt\pi \left(\frac{z_1 z_2 e^2}{8\pi\epsilon_0 K}\right)^2 \cot(\theta/2)\csc^2 \frac{\theta}{2}$$

N_i: INCIDENT PARTICLES PER SEC.

PARTICLES SCATTERED INTO RING $d\theta$ = $N_i |df|$

$$N(\theta) = \frac{\text{Particles}}{\text{AREA}} = \frac{N_i \, |df|}{2\pi r^2 \sin\theta \, d\theta}$$

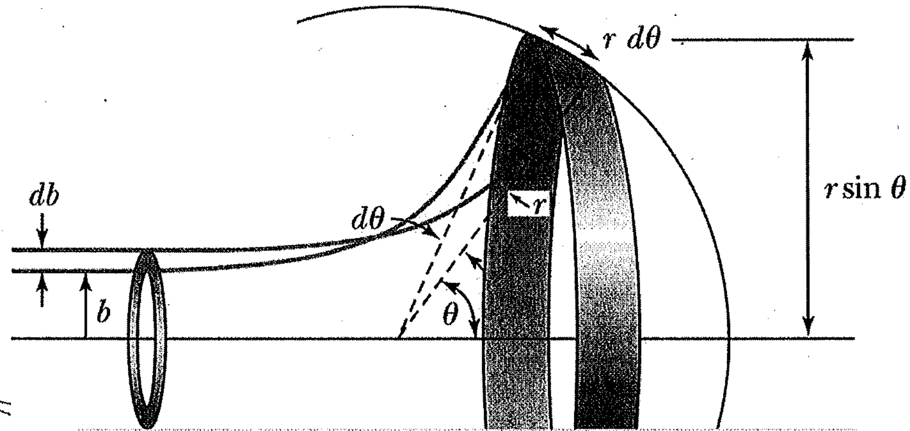
$$= \frac{N_i \, n t \pi \left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K}\right)^2 \cot(\theta/2) \csc^2\frac{\theta}{2} \, d\theta}{2\pi r^2 \sin\theta \, d\theta}$$

$$= \frac{N_i n t}{8} \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{Z_1^2 Z_2^2}{r^2 K^2} \frac{\cot\frac{\theta}{2} \csc^2\theta/2}{\sin\theta}$$

$$\frac{\cos\theta/2}{\sin\theta/2} \frac{1}{\sin^2\theta/2} \left(\frac{1}{2\sin\theta/2 \cos\theta/2}\right)$$

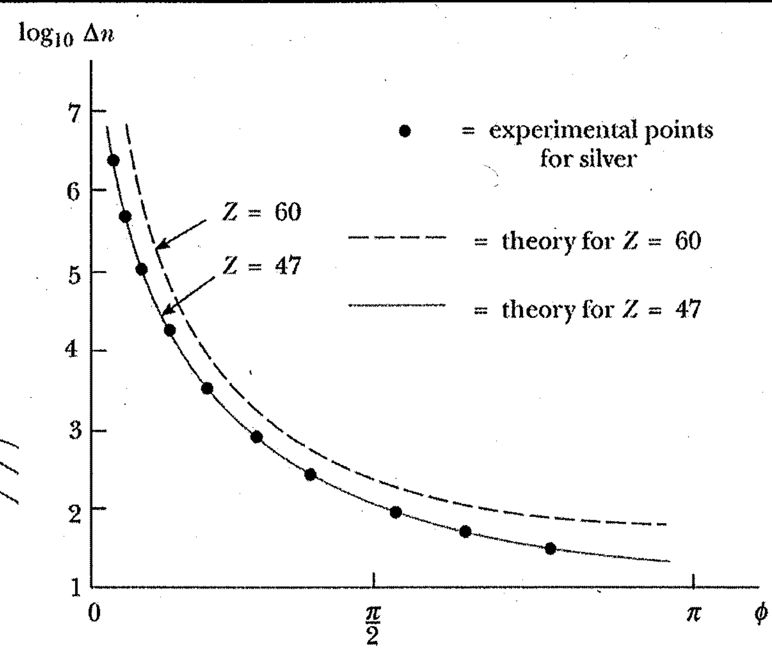
$$= \frac{1}{2\sin^4\theta/2}$$

= RUTHERFORD SCATTERING FORMULA.



$$N(\theta)=\frac{N_i nt}{16}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{Z_1^{\,2}Z_2^{\,2}}{r^2(K)^2\sin^4\left(\frac{\theta}{2}\right)}$$

Image from Thornton and Rex, Modern Physics, 4th ed. Brooks Cole



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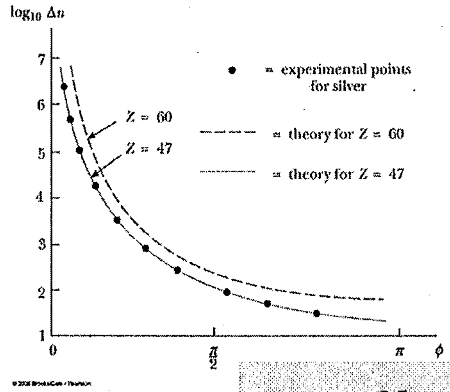
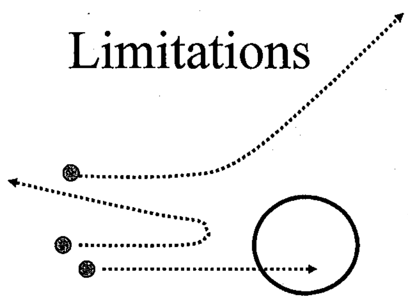


$$N(\theta)=\frac{N_i nt}{16}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{Z_1^2 Z_2^2}{r^2 (K)^2 \sin^4\left(\frac{\theta}{2}\right)}$$

$$\frac{e^2}{4\pi\varepsilon_0}=1.44\, eV \cdot nm$$

Limitations



- If K_α is large enough, it can breach the nucleus. In that case Rutherford's formula fails.

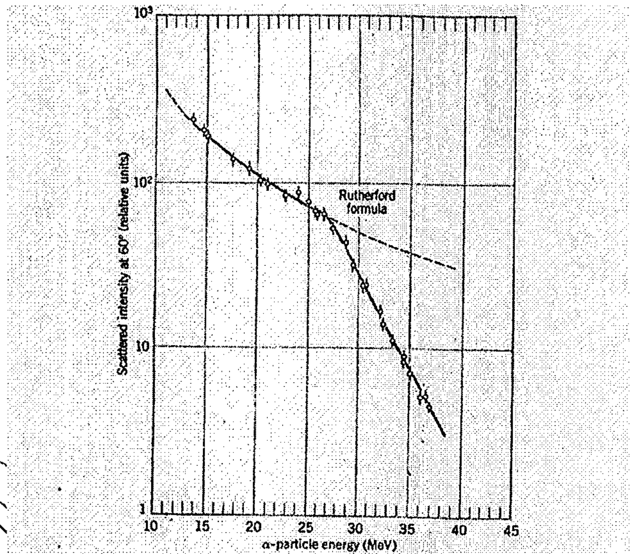


Figure 3.11 The breakdown of the Rutherford scattering formula. When the incident α particle gets close enough to the target Pb nucleus so that they can interact through the nuclear force (in addition to the Coulomb force that acts when they are far apart) the Rutherford formula no longer holds. The point at which this breakdown occurs gives a measure of the size of the nucleus. Adapted from a review of α particle scattering by R. M. Eisberg and C. E. Porter, *Rev. Mod. Phys.* **33**, 190 (1961).

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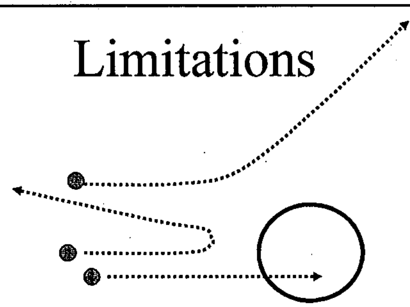
Limitations



- If K_α is large enough, it can breach the nucleus. In that case Rutherford's formula fails.
- Used to determine nuclear radius!

$$K = \tfrac{1}{2} m_\alpha {v_\alpha}^2 = \frac{1}{4\pi\varepsilon_0} \frac{Z_1 e (Z_2 e)}{d_{\min}}$$

$$\frac{e^2}{4\pi\varepsilon_0} = 1.44\, eV \cdot nm$$

Discove

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$$K = \frac{e^2}{4\pi\varepsilon_0}\frac{Z_1(Z_2)}{d_{min}}$$

$$d_{min} = \frac{e^2}{4\pi\varepsilon_0}\frac{Z_1(Z_2)}{K}$$

$$d_{min} = (1.44 eV \cdot nm)\frac{2(82)}{27\times10^6 eV}$$

$$d_{min} = 8.8\times10^{-6} nm$$

Moral: Geiger and Marsden experiment agrees with Rutherford's Model:

Dense, positive nucleus at center of the atom

- *diameter of an atom* $\leq 3\times10^{-10}$ m
- *diameter of a nucleus* $\leq 10\times10^{-15}$ m

Radius of atomic nuclei

$$r = r_0 A^{1/3}$$

$$r_0 = 1.2\ fm$$

Demo!

Alpha tracks in a cloud chamber

Observation – mass of nucleus (A) is just about twice atomic number (Z)

Possible (wrong) model for Helium 4:

4 protons and 2 e^-'s

Recall deBroglie Wavelength

$$\lambda = \frac{h}{p}$$

For an e^- confined in a few *fm*, kinetic energy is hundreds of MeV

(relativistic calculation!)

No mechanism for such large energies

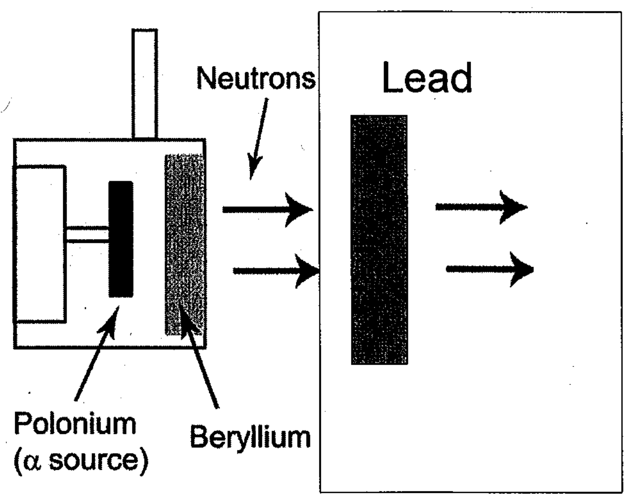
This and other considerations (spin, nuclear magnetic moment) suggest there are no e^- 's in the nucleus.

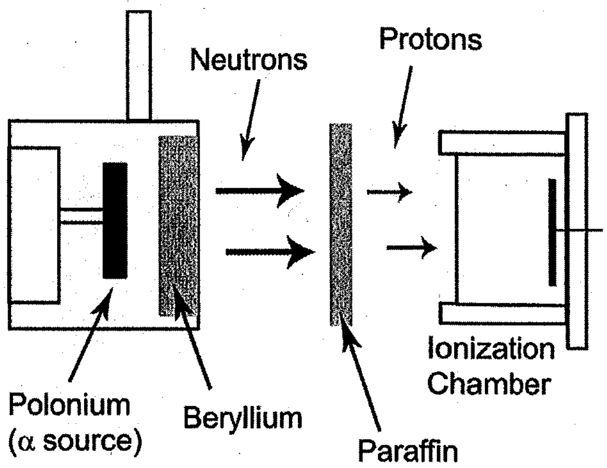
1932 – James Chadwick "discovers" the neutron

$$ {}^{4}_{2}He + {}^{9}_{4}Be \rightarrow {}^{12}_{6}C + n $$

http://www.nobelprize.org/nobel_prizes/physics/laureates/1935/chadwick-facts.html

E and *B* fields have no (um…little) effect on the neutron





1932 – Irene and Frederic Curie-Joliot measured 5.7 MeV protons

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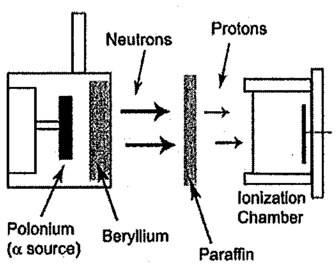

^{4}He + Be produces uncharged radiation

Could it be a gamma ray?

- The gamma ray could Compton scatter protons from the paraffin
- Such a gamma ray would have to be 50 MeV in energy – too much energy

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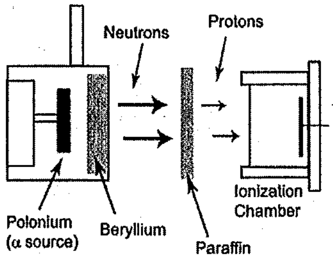
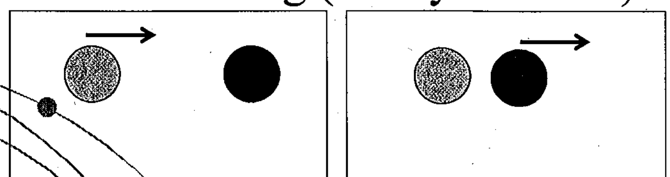


Chadwick assumed neutral particles with mass near mass of a proton

Elastic scattering (nearly head-on)



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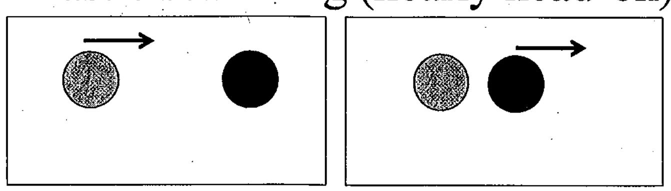
TOP HAT

Elastic scattering (nearly head-on)



For this head-on elastic collision between objects of the same mass…

a.) $KE_{green} > Ke_{red}$

b.) $KE_{green} < KE_{red}$

c.) $KE_{green} = KE_{red}$

$KE\ of\ n \approx KE\ of\ p$

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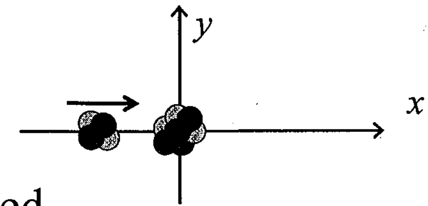
$${}^{4}_{2}He + {}^{9}_{4}Be \rightarrow {}^{12}_{6}C + n$$



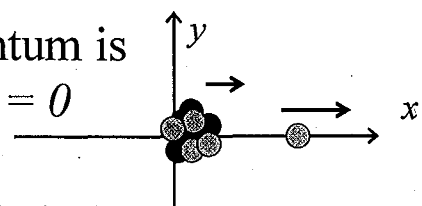
Mass-energy is conserved…

$$K_{\alpha} + m_{\alpha}c^2 + m_{Be}c^2 = K_C + m_C c^2 + K_n + m_n c^2$$

x-component of momentum is conserved…

$$p_{\alpha x} = p_{Cx} + p_{nx}$$

y-component of momentum is conserved..Assume $p_{ny} = 0$



m_n can be determined

Feb., 1932 – "The Possible Existence of a Neutron"

May, 1932 – "The Existence of a Neutron"

$$m_n c^2 = 939.566\ MeV$$

$$m_p c^2 = 938.272\ MeV$$

$$m_e c^2 = 0.511\ MeV$$

Nuclear Structure and Properties

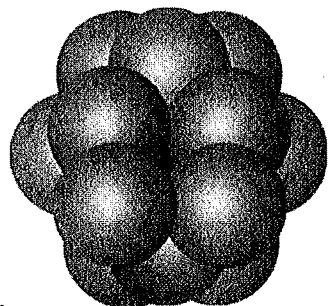
Atomic Nuclei
– collection of protons and neutrons
- nucleons

$^{A}_{Z}X$

Z – atomic number
number of protons

N – neutron number
number of neutrons

A – mass number
$A=Z+N$



Isotopes
– nuclei with same Z and different A and N

example $^{11}_{6}C$ $^{12}_{6}C$ $^{13}_{6}C$

Why is the nucleus bound at all?

^{4}He

$r = 1.2\ fm\ A^{1/3}$ $\quad r = 1.90 \times 10^{-15}\ m$

Coulomb repulsion of the protons:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$= \left(9 \times 10^9 N \frac{m^2}{C^2}\right)\frac{\left(1.6 \times 10^{-19} C\right)^2}{\left(1.9 \times 10^{-15} m\right)^2} = 64\ N$$

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Why is the nucleus bound at all?

^{4}He

64 N doesn't sound so big…

Gravitational attraction of the protons:

$$F = G\frac{m_1 m_2}{r^2}$$

$$= \left(6.67 \times 10^{-11} N \frac{m^2}{kg^2}\right)\frac{\left(1.67 \times 10^{-27} kg\right)^2}{\left(1.9 \times 10^{-15} m\right)^2}$$

$$= 5.15 \times 10^{-35}\ N$$

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Why is the nucleus bound at all?

$F_{electric}$ repulsion is HUGE!

^{4}He

New force needed to bind nucleons

Strong Nuclear Force

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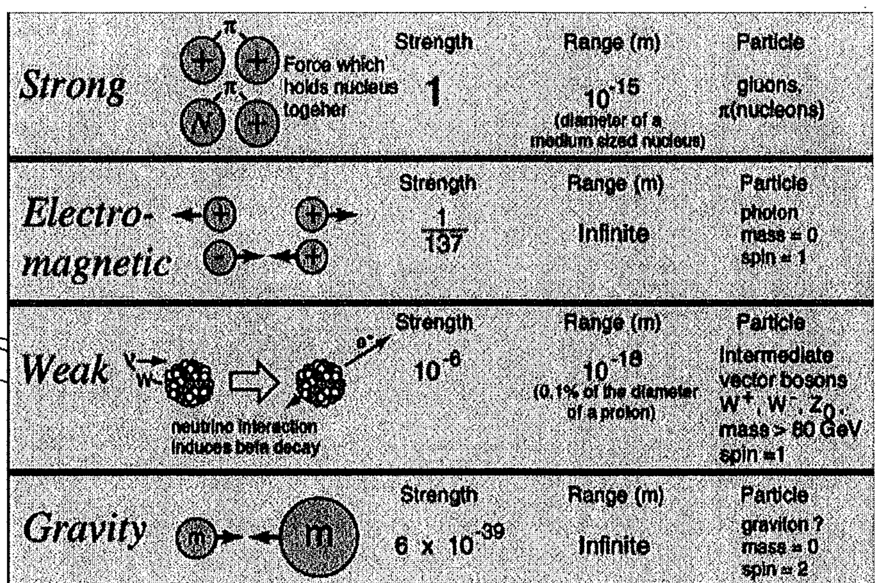
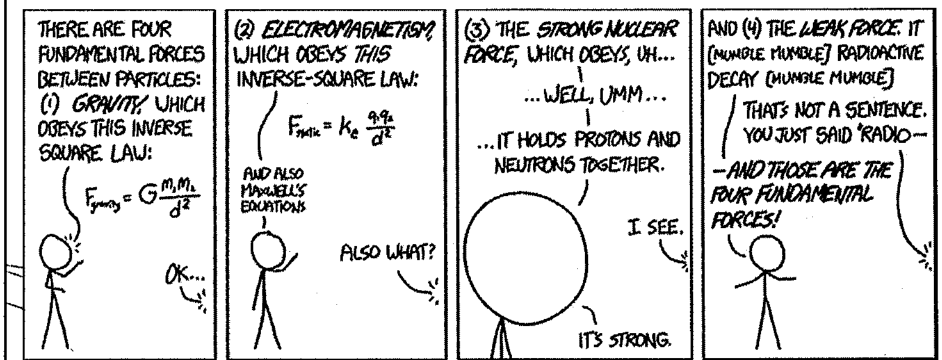
Four Fundamental Forces



Force		Strength	Range (m)	Particle
Strong	Force which holds nucleus together	1	10^{-15} (diameter of a medium sized nucleus)	gluons, π(nucleons)
Electro-magnetic		$\frac{1}{137}$	Infinite	photon mass = 0 spin = 1
Weak	neutrino interaction induces beta decay	10^{-6}	10^{-18} (0.1% of the diameter of a proton)	Intermediate vector bosons W^+, W^-, Z_0, mass > 80 GeV spin = 1
Gravity		6×10^{-39}	Infinite	graviton ? mass = 0 spin = 2

http://alpcentauri.info/fundamental_forces.html



http://imgs.xkcd.com/comics/fundamental_forces.png

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Strong Nuclear Force

- Strong at short distances
- Negligible above atomic distances
- Only affects some particles (not e^-)

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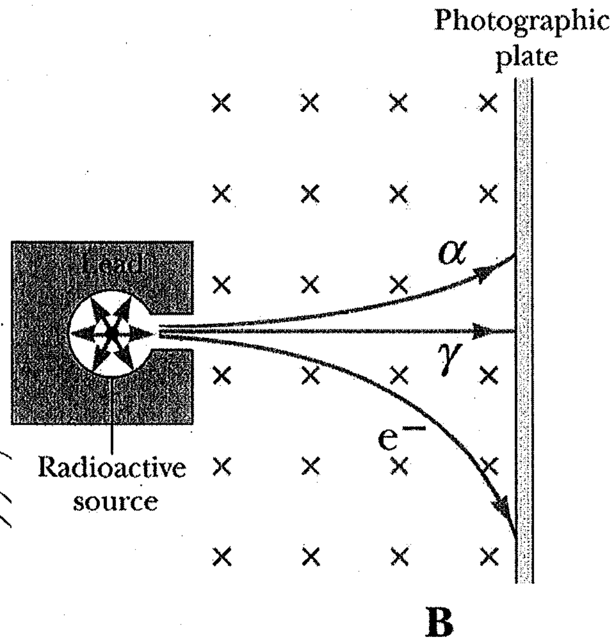
Radioactivity

Discovered by Henri Becquerel (1896)

Spontaneous emission of radiation by atomic nuclei

Three types of radioactivity

Alpha (α)	Beta (β)	Gamma (γ)
^{4}He nuclei	*e^-'s or e^+'s*	High E photons
	positron *(e^+)*	



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Shielding

α – stopped by a few sheets of paper

β – stopped by a few mm of Aluminum

γ – stopped by cm of lead

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Alpha (α)

$$^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}He$$

Parent nuclei

Daughter nuclei

Applications – smoke detectors, pacemakers, Mars exploration…

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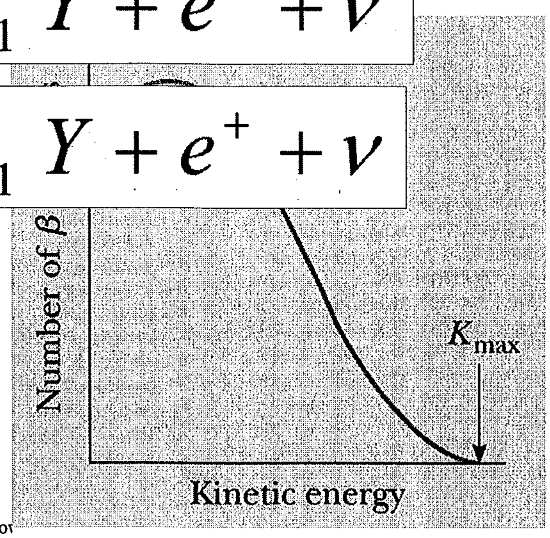
Beta (β)

$$^{A}_{Z}X \rightarrow ^{A}_{Z+1}Y + e^{-} + \overline{\nu}$$

$$^{A}_{Z}X \rightarrow ^{A}_{Z-1}Y + e^{+} + \nu$$



Applications – carbon dating…

Discov

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^{-} + \overline{\nu} $$

Anti-neutrino

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^{+} + \nu $$

Neutrino

Gamma (γ)

$$ {}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma $$

Applications –Sterilizing by irradiation, CT scans, …

Nuclear decay rate

$$\frac{dN}{dt} = -\lambda N$$

- $\frac{dN}{dt}$: Rate of change in number of radioactive nuclei
- λ: Decay constant
- N: Number of radioactive nuclei in a sample

$$\frac{dN}{dt} = -\lambda N$$

$$\frac{dN}{dt} < 0$$ N is decreasing

$$\frac{dN}{N} = -\lambda dt$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

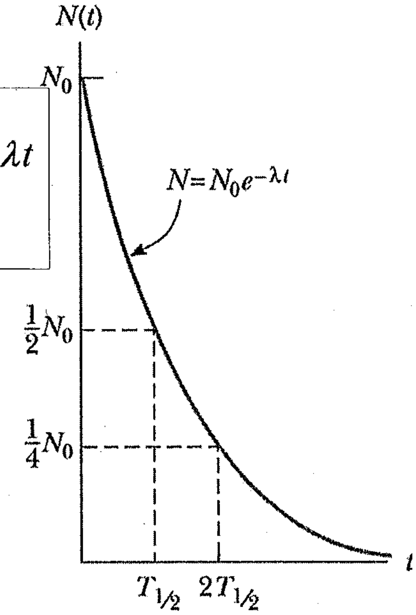
$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$N = N_0 e^{-\lambda t}$$

Decay rate (Activity)

$$R = \left|\frac{dN}{dt}\right| = N_0 \lambda e^{-\lambda t}$$

$$R = R_0 e^{-\lambda t}$$



Half-life ($T_{1/2}$)

$$N = N_0 e^{-\lambda t}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$2 = e^{+\lambda T_{1/2}}$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

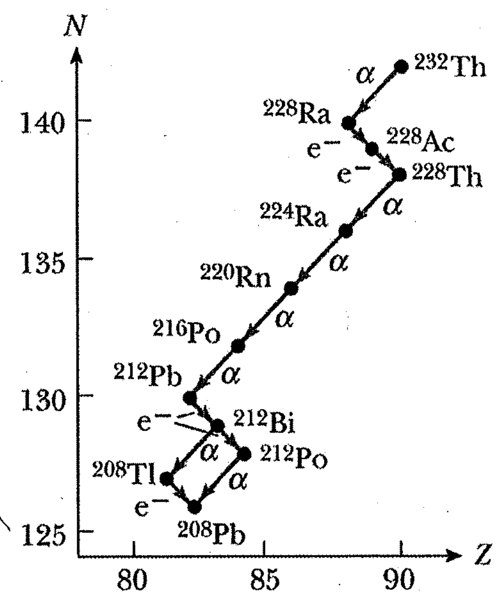
$$T_{1/2} = \frac{0.693}{\lambda}$$

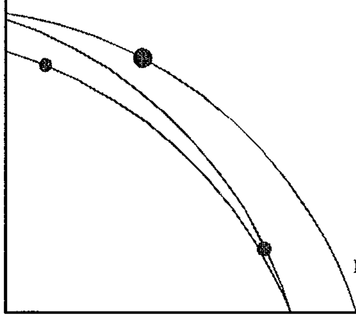
TOP HAT

The mean life of a particular isotope is *84 hr*. The initial activity of this isotope at $t = 0$ is $68\mu Ci$. How many nuclei will decay in the time interval from $t_1 = 3\ hr$ to $t_2 = 9\ hr$?

$$1\,Ci = 3.7\times10^{10}\,decays/\sec$$

5.06 e+10





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