Math 380: "Algebraic Number Theory"

Catalog Description:

Number Theory is the study of discrete number systems such as the integers. When studying integer solutions to a polynomial equation one is led to work with the more general "algebraic numbers." Studying algebraic number fields leads to a deep understanding of units and factorization, which are important in both High School and research mathematics. Zeta functions give information in the form of generating functions for the key properties of the number fields in question. Understanding the Riemann Hypothesis, a conjecture with a solution worth one million dollars, gives a glimpse of the forefront of research mathematics.

Prerequisites:

Math 319 or Math 330.

Purpose:

Algebraic number theory is an excellent next step for students who have previously studied abstract algebra (math 330). A course in algebraic number theory can provide an opportunity to reinforce much of the material from abstract algebra, and give students a deeper understanding of how some of those ideas are used in higher mathematics.   In addition, the course is a natural sequel to the theory of numbers (math 319), as it dives much deeper into many of the same topics.

Abstract algebra has recently become a requirement for majors in Mathematics with Certification in Secondary Education. Whereas many universities offer an abstract algebra 2 course geared towards students planning to attend graduate school, algebraic number theory is a more ideal course for prospective secondary education teachers because of its emphasis on a deep understanding of prime numbers and their factorizations. For this reason, some universities (such as our sister college, The College of New Jersey) require a number theory course for prospective High School Teachers. For those on the graduate school track an early look at advanced topics, such as the Clay Mathematics Institute's million dollar Riemann hypothesis, is invaluable to motivation and mathematical maturity.

Evaluation:

1. Assignments: 40%
2. Student projects: 20%
3. Midterm exam: 20%
4. Final exam: 20%

Topics:

I. Representation of primes by quadratic forms
II. Rings and Ideals
III. Quadratic number fields
IV. Dirichlet unit theorem
V. Class numbers
VI. Dedekind zeta function
VII. Riemann zeta function
VIII. Riemann hypothesis

Learning objectives:

1. Students will gain an understanding of the relation between abstract algebra and number theory.
2. Students will improve their understanding of units and learn the general structure of the unit group for the ring of integers of quadratic number fields.
3. Students will improve their understanding of factorization and learn how class numbers measure the degree of failure of unique factorization.
4. Students will learn how generating functions and zeta functions encode a large amount of information into one mathematical object.
5. Students will see cutting edge mathematics via student projects and class discussion of the Riemann Hypothesis.
6. Students will come to appreciate the need for further studies in algebra and number theory.

Course resources:

1. H. Cohn, Advanced Number Theory
2. D. Cox, Primes of the form $x^2 + n y^2$
3. K. Ireland, M. Rosen, A Classical Introduction to Modern Number Theory
4. W. LeVeque, Topics in Number Theory
5. D. Marcus, Number Fields
6. M. Newman, Integral Matrices
7. P. Ribenboim, Classical Theory of Algebraic Numbers
8. P. Samuel, Algebraic Theory of Numbers
9. I. Stewart, D. Tall, Algebraic Number Theory and Fermat's Last Theorem
10. J. Stillwell, Elements of Number Theory