Math 380: Applied and Computational Linear Algebra
(either as a topics course or a course that could count as our "discrete" option)

Linear Algebra is arguably one of the most useful areas of mathematics, encompassing the theoretical and applied, and in particular in applied math, computational math, statistics, physics, engineering, bio-mathematics, and many other areas. This course will cover some of the many applications that are typically not covered in other math courses, and will cover what is necessary to effectively utilize the mathematics and coding necessary to implement these applications.

Pre-requisites: Math 233, Math 230 or permission of instructor

Motivation

Math 233 Linear Algebra 1 gives students a foundation in the basics of linear algebra, and is often restricted to only considering the vector space $\mathbb{R}^n$. Math 333 Linear Algebra 2 extends to complex vector spaces, $\mathbb{C}^n$, complex *mxn* matrices, function spaces, like $\mathrm{T}^n = \mathrm{span}\left\{e^{ikt}\right\}_{k=-n}^{n}$, P_n the set of polynomials of degree less than or equal to *n*, and $C^{(n)}(a,b)$, the set of *n* continuously differentiable functions on the interval *(a,b),* as well, as the more theoretical foundations of linear algebra.

There is insufficient time in either course to explore the many applications of linear algebra, and none to truly explore the computational side, as Math 230 is not a prerequisite of either course.

This course will fill in the gaps by reviewing/introducing only as much linear algebra and coding is needed in order to explore these applications.

Topics

A variety of topics will be selected from

- orthogonal projections
    - least squares
    - image processing, tomography and medical resonance imaging
- Singular Value Decomposition (SVD)
    - data reduction with Principal Component Analysis (PCA)
    - image compression
- similarity transformations
    - Hessenberg form
    - Householder transformations
    - orthogonal diagonalization
    - Jordan Canonical Form
    - Wavelet transformations
- numerical computations of eigen values and eigen vectors
    - power method
    - Krylov subspaces
    - QR-algorithm
- numerical optimization
    - Conjugate Gradient Method, Lanczos Method, and GMRES (generalized minimal residual method)
- computational topics from linear programming, and

- machine learning
    - Neural Networks

Learning Outcomes
Upon successful completion of the course students will be able to

- distinguish between analytical and numerical methods
- apply and analytically solve problems that utilize linear algebra
- define/derive and code algorithms to numerically solve problems that utilize computational linear algebra
- analyze the error in numerical computations and compare different approaches of numerical solutions
- complete a group project that utilizes the math and programming from the course and present the results in lieu of a final examination

Bibliography

Applied and Computational Linear Algebra: A first course, Charles Byrne
Applied Linear Algebra, Noble and Daniel
Elementary Linear Algebra (Applications version), Anton and Rorres
Introduction to Linear Algebra, Johnson, Riess, and Arnold
Linear Algebra and Learning From Data, Strang
Linear Algebra with Applications, Leon
Matrix Computations, Golub and Van Loan
Numerical Linear Algebra, Trefethen and Bau