

To the Student:

Welcome to ProofSpace!

There's something uniquely satisfying about completing a mathematical proof. It's comparable to solving a mystery, cracking a code, or figuring out a puzzle. For many of you, this kind of mathematics will bear little resemblance to the math you saw in high school. We hope many of you think of what you see here as an extended puzzle, and that you find something fun in the process.

Pure Mathematics is fundamentally concerned with fascinating concepts – truth, validity, infinity, distance, shape, and categorization. We hope you will use the vocabulary you learn in this space to provide a new sort of approach to the world around you.

If you're using this space informally: You can use ProofSpace on your own in various ways: as an all-in-one introduction to the world of mathematical proofs, as a companion resource to use alongside other resources, or as a review of material that you have already learned. We hope that the videos here provide you with a good refresher or alternative viewpoint. We also recommend using the comprehension quizzes and problem sets to guide your learning.

If you're using this space for a class: There are quite a few advantages to learning how to do mathematical proofs in this video/problem format, but perhaps the most important to you is that you can really go at your own pace. Try to make a habit of pausing the videos and trying the problems presented in the videos. Maybe you'll want to try the quiz after watching the videos, or maybe you'll want to take a look at it beforehand. In this format, you will have a huge say in how you learn the material. We trust you to be the best judges of your own pace and method – but if something isn't working for you, talk to a professor or classmate about other ways you might approach the videos. *Regardless of how you approach this course, you should definitely take notes while watching the videos. The definitions and theorems will be used and reused throughout the course.*

For a greater chance of success, we recommend a four (or five) tier learning process:

1. Watch the videos and take notes,
2. Take the Comprehension Quiz,
3. Work through the Discussion Problems with others,
4. Work through the Evaluated Problems without guidance, and
5. (Optional) Get extra practice with the Supplemental Problems.

Each chapter is split up into various sections. (1) For each section, start by watching the **videos** and taking notes the way you would in a typical lecture-style course. (2) This should be followed up by taking the **Comprehension Quiz**, which is made up of basic questions and computational problems associated to the topics of the videos. These first two steps should be done prior to your class meeting. (3) Once you feel like you have a solid foundational understanding of the basics, it's time to go to work on the **Discussion Problems**. Ideally, these problems are discussed and completed during class, working individually or in small groups, under the guidance of an instructor, teacher, or professor. You might also work on them in small groups of students outside of class. They are intended to be discussed with others to set you up for the next step. (4) Now you should be ready to work on the **Evaluated Problems** on your own. Of course you are free to work on these problems with other students, but the goal is to be able to complete them without the guidance of your instructor. These problems are similar to the

discussed problems, often directly related extensions of them. (5) This final step is optional, but mathematical understanding, retention, and success is most often attained through practice and repetition. Whether or not you feel comfortable with your level of understanding of the topics covered in each section after you complete the discussed and evaluated problems, it's usually a good idea to try some of the **Supplemental Problems**. These are provided to you as extra practice problems to help you gain a stronger understanding of the topics covered in each section.

An Extra Resource for Reading: Some students learn better from reading about a topic rather than watching videos. Perhaps surprisingly, many students don't actually know how to read mathematics. The ability to READ about mathematics is a highly important skill that must be learned. Reading mathematics is not like reading a novel. It is technical reading that must be done with patience, care, and thoughtfulness. An excellent resource for you as you work your way through ProofSpace is Dr. Ted Sundstrom's *Introduction to Mathematical Reasoning and Proof*, which you can find online here:

<https://sites.google.com/site/mathematicalreasoning3ed>.

Our presentation is different from Dr. Sundstrom's, especially in the earlier chapters, but as a free online resource, it's a great second approach. If you use this book, please **use the definitions from the videos and not from the book** when working on the problems. The supplemental problems in each problem set refer to the online version 2.0 of this book.

A note about the advanced problems: The advanced problems serve a variety of purposes. Sometimes, they're tangentially related puzzles. Other times, they walk you through extensions of the work at hand. Still other times, they introduce a brand new concept that you might see later on in mathematics. Even if you haven't tried the advanced problems previously, we encourage you to look at them for each individual problem set. Sometimes they may be more to your taste than other times.

We encourage feedback! Please provide us with questions, comments, concerns, constructive criticisms, and compliments by e-mailing us at heap@geneseo.edu.

The reason we put a lot of time and energy into the creation of this space is because we think there's something really valuable about the proofs skill set for students of all backgrounds and aims. We hope that you find this to be true for you.

Good luck, and happy proving!

~ The ProofSpace Team