

Theorem (Summary of Useful Logical Equivalencies)

De Morgan's Law:

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

Contrapositive:

$$P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$$

Conditional as a Disjunction:

$$P \Rightarrow Q \equiv \neg P \vee Q$$

Negation of a Conditional:

$$\neg(P \Rightarrow Q) \equiv P \wedge \neg Q$$

Double Negation:

$$\neg(\neg P) \equiv P$$

Idempotent:

$$P \vee P \equiv P \equiv P \wedge P$$

Commutativity:

$$P \wedge Q \equiv Q \wedge P$$

$$P \vee Q \equiv Q \vee P$$

Associativity:

$$(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$$

$$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$$

Distribution:

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

Disjunction with a Contradiction:

$$\text{If } C \text{ is always false, then } P \vee C \equiv P$$

Conditional with a Disjunction:

$$P \Rightarrow (Q \vee R) \equiv (P \wedge \neg Q) \Rightarrow R$$

$$(P \vee Q) \Rightarrow R \equiv (P \Rightarrow R) \wedge (Q \Rightarrow R)$$