ProofSpace Problem Set

Sets

Introduction

Discussed Problems

1 One of the most important aspects of basic set theory is interpreting set notation in natural language and being able to determine the elements in the set. For each of the following sets, interpret the set builder notation by listing out the elements of the set.

a) $\{a \in \mathbb{N} \mid (\exists y \in \mathbb{N})(ay = 4)\}$

b) $\{m \in \mathbb{Z}$ such that $m^2 - 2m - 1 = 0\}$

c) $\{m \in \mathbb{R} \mid m^2 - 2m - 1 = 0\}$

d) $\{p \in \mathbb{Z} \mid 10 < p^2 < 100\}$

e) $\{x \in \mathbb{Q} \mid \sqrt{x+2} = \frac{4}{3}\}$

f) $\{y \in \mathbb{Z}$ such that 3 divides $(y + 1)\}$

g) $\{y \in \mathbb{R} \mid 3 \text{ divides } (y + 1)\}$

2 Likewise, it's important to be able to translate from roster notation or natural language into good set builder notation. For each of the following, write the set in set builder notation.

a) The set of all real solutions to the equation $x^2 - 2x - 1 = 0$.

b) The set of all irrational integers.

c) $\{... - 7, -1, 5, 11, 17, ...\}$

d) $\{5, 11, 17, 23, ...\}$

e) The set of all natural numbers for which there exists an integer such that the natural number and the integer added together are less than 8.

3 For each of the following, give an example of a set B for which the statement is true, if such a set exists. Decide if the statement is true for all B, some B, or no B.

a) $B \in \mathcal{P}(B)$.

b) $\{B\} \subseteq \mathcal{P}(B)$.

c) $B \subseteq \mathcal{P}(B)$.

d) $B = \mathcal{P}(B)$.

e) $B \not\subseteq \mathcal{P}(B)$.

Evaluated Problems

1 For each of the following sets, interpret the set builder notation by listing out the elements of the set.

a) $\{x \in \mathbb{Z} \mid (3 \text{ divides } x \text{ and } 3 \text{ divides } x^2)\}$.

b) $\{n \in \mathbb{N} \mid (\forall m \in \mathbb{N})(n + m > 4)\}$

c) $\{p \in \mathbb{Z} \mid (p^2 < 0) \Rightarrow (p = 4)\}$.

2 For each of the following, write the set in set builder notation.

a) The set of integers y such that for all integers x, $(x + y)$ is an integer.

b) The set of all real numbers p such that there exists a real number q such that for all real numbers m, $qm > p$.

c) Recall that a prime number is any natural number with exactly two positive factors, 1 and itself. Express the set of prime numbers in set builder notation.

3 Let $A = \{\emptyset, \{\emptyset\}, \emptyset\}$. What is $\mathcal{P}(A)$?

Supplemental Problems

Mathematical Reasoning: Writing and Proof, Online Version 2.0, by Ted Sundstrom:
Sec. 2.3: 1, 2, 4, 5
Sec. 5.1: 1, 2, 4

Advanced Problems

1 This is not a difficult problem. The point of it is just to give you a chance to explore the relationship between the number of elements in a set and the number of elements in its power set. Let $A = \{2, 3, \{2\}, \mathbb{Z}\}$ and $B = \{\emptyset, \{\mathbb{Z}\}, 4\}$. How many elements are in each of the following sets?

a) A

b) B

c) $\mathcal{P}(A)$

d) $\mathcal{P}(B)$

2 Let A be a finite set of n elements. Make a conjecture about the size of $\mathcal{P}(A)$ in terms of n. Then, prove your conjecture. HINT: There are many ways to approach this problem. For one method, think about how you might prove something for all $n \in \mathbb{N}$.