ProofSpace Comprehension Quiz

Proof Techniques

Induction

1 Which of the following is the **most appropriate** way to end a proof by induction?

(a) Since $P(k)$ implies $P(k+1)$, the proposition is true by induction.

(b) Since $P(k)$ implies $P(k+1)$, and $P(1)$ holds, the proposition is true by induction.

(c) Since $P(k)$ implies $P(k+1)$ for all $k > 1$, and $P(1)$ holds, the proposition is true by induction.

(d) Since $P(1), P(k)$ and $P(k+1)$ all hold, the proposition is true by induction.

(e) Since the induction hypothesis holds, the proposition is true by induction.

2 Rank the steps below in the order you would do them in a proof by induction, where "1" is the first step and "5" is the last step.

a) We will show $P(1)$ is true (the base case).

b) We will use what we've established to show $P(k+1)$ is true.

c) We will state that we are using a proof by induction.

d) We will assume $P(k)$ is true.

e) We will conclude the proof.

3 Consider the following proof by induction.

Theorem: For all natural numbers, $1 + 2 + ... + n = \frac{(n)(n+1)}{2}$.

Proof: We will prove the proposition by induction on n. Let $P(n)$ be the statement

$$1 + 2 + ... + n = \frac{(n)(n+1)}{2}.$$

Observe that

$$1 = \frac{2}{2} = \frac{(1)(2)}{2} = \frac{1(1+1)}{2},$$

so clearly $P(1)$ holds. Now, assume that $P(r)$ holds for some r, that is, that

$$1 + 2 + ... + r = \frac{(r)(r+1)}{2}.$$

Then,

$$1 + 2 + ... + r + (r+1) = \frac{(r+1)(r+2)}{2}.$$

So by the induction hypothesis,

$$\frac{(r)(r+1)}{2} + (r+1) = \frac{(r+1)(r+2)}{2}.$$

$$\frac{(r)(r+1)}{2} + \frac{2(r+1)}{2} = \frac{(r+1)(r+2)}{2}.$$

$$\frac{(r+1)(r+2)}{2} = \frac{(r+1)(r+2)}{2}$$

which is true, so therefore $P(r+1)$ is true. Now, (use the concluding sentence you selected in question 1). □

Which of the following **best** explains the **most significant error** of this proof?

(a) Did not label the assumption as the induction hypothesis.

(b) Assumed $P(1)$

(c) Assume $P(r+1)$ and worked backwards to show it was true.

(d) Used r instead of k.

(e) The proof had no significant errors.