ProofSpace Comprehension Quiz

Sets

Proving Subsets and Set Equality

1 This item is a bit challenging; we recommend scrap paper. For each set in Column A, choose the appropriate equivalent set from Column B. Each entry in Column B may be used no more than once.

Column A	**Column B**
____**1)** $\{x \in \mathbb{R} \mid x^2 - 2 = 0\}$	**a)** $\emptyset$
____**2)** $\{x \in \mathbb{Q} \mid x^2 - 2 = 0\}$	**b)** $\{\sqrt{2}, -\sqrt{2}\}$
____**3)** $\{x \in \mathbb{R} \mid x(x+1)(x+2) = 0\}$	**c)** $\{0, -1, -2\}$
____**4)** $\{x \in \mathbb{N} \mid \exists y \in \mathbb{N}(x + y > 4)\}$	**d)** $\{... - 5, -4, 4, 5, 6...\}$
____**5)** $\{x \in \mathbb{N} \mid \forall y \in \mathbb{N}(x + y > 4)\}$	**e)** $\{4, 5, 6...\}$
	f) $\mathbb{N}$
	g) $\mathbb{R}$

2 Let $A = \{x \in \mathbb{N} \mid x^2 < 100\}$ and $B = \{x \in \mathbb{Z} \mid 4 \text{ divides } x\}$. Compute the following:

a) $A \cup B$

b) $A \cap B$

3 Consider the following proof that two sets are equal.

Theorem: For any sets A, B, and C,

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Proof: Let x be an element of $A \cup (B \cap C)$. Then, $x \in A$ or $x \in B \cap C$. Suppose $x \in A$. Then, $x \in A \cup B$ and $x \in A \cup C$. Thus, $x \in (A \cup B) \cap (A \cup C)$ and $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.

Now, let y be an element of $(A \cup B) \cap (A \cup C)$. Then, $y \in A \cup B$ and $y \in A \cup C$. So, $y \in A$ or $y \in B$ and $y \in A$ or $y \in C$. Suppose $y \in A$. Then, $y \in A \cup (B \cap C)$, as desired. Now, suppose $y \notin A$. Then, since $y \in A$ or $y \in B$, it must be that $y \in B$. Similarly, since $y \in A$ or $y \in C$ and $y \notin A$, it's true that $y \in C$. Thus, $y \in B \cap C$ and $y \in A \cup (B \cap C)$. Thus, $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$.

Since $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ and $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$, it's true that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. □

Which of the following **best** explains the **most significant error** of this proof?

(a) Did not prove both containments.

(b) Used an unproved fact.

(c) Did not consider if $y \notin B$.

(d) Assumed $x \in A$.

(e) The proof had no significant errors.