ProofSpace Comprehension Quiz

Functions

Composition and Inverses

1 Consider the following functions:

$f : A \to B$
$g : B \to C$
$h : C \to A$
Assume each function is a bijection.

For each of the following, decide if the entry given is a function. If it is, write down its domain and codomain.

a) f^{-1}

b) g^{-1}

c) h^{-1}

d) $f(h(x))$

e) $g \circ f$

f) $h \circ f$

g) $f \circ f$

h) $h(g(x))$

i) $g \circ h$

j) $g \circ g$

k) $g \circ g^{-1}$

l) $g \circ f^{-1}$

m) $g^{-1} \circ f^{-1}$

n) $h^{-1} \circ f$

2 Consider the following proof.

> **Theorem:** Let $f : A \to B$ and $g : B \to C$ be surjections. Then, $g \circ f$ is a surjection.
> **Proof:** Let $y \in C$. Then, since g is surjective, there is an $z \in B$ such that $g(z) = y$. Since $z \in B$ and f is surjective, there exists an $x \in A$ such that $f(x) = z$. Thus, since $g(z) = y$, by substitution, $g(f(x)) = y$. Observe, therefore, that for any $y \in C$ there exists an $x \in A$ such that $g(f(x)) = y$. Therefore, $g \circ f$ is surjective, as desired. □

Which of the following **best** explains the **most significant error** of this proof?

(a) Used the wrong codomain.

(b) Used the wrong domain.

(c) Used an unproven fact.

(d) Failed to take into account $f \circ g$.

(e) The proof had no significant errors.