ProofSpace Comprehension Quiz

Relations

Equivalence Classes

Questions 1 and 2 are designed to help you think intuitively about equivalence classes.

1 Consider the natural language relation on cities in the United States given by

> $(x, y) \in R$ if and only x and y are in the same state.

This is an equivalence relation. Which best represents its equivalence classes?

(a) The United States of America

(b) The set of States

(c) The collection of cities in New York

(d) All cities in the United States

2 Consider the natural language relation on the set of math classes at Geneseo this semester given by

> $(x, y) \in R$ if and only if x and y are taught by the same professor.

This is an equivalence relation. Which best represents its equivalence classes?

(a) The set of all math classes at Geneseo this semester

(b) The set of all professors

(c) The set of all classes taught by Dr. Jones

(d) The set of all calculus classes

3 Consider the equivalence relation on $A = \{1, 2, 3, 4, 5\}$ given by

$$R = \{(1,1),(1,2),(1,3),(2,2),(2,3),(2,1),(3,3),(3,1),(3,2),(4,4),(4,5),(5,5),(5,4)\}$$

Clearly, $[1] = [2]$.

a) How many equivalence classes does this relation have? _____

b) Decide if the following statements are true or false:

(a) $[1] = [3]$. _____
(b) $[4] = [3]$. _____
(c) $[5] = [3]$. _____
(d) $[4] = [5]$. _____
(e) $[2] = [3]$. _____
(f) $[2] = [4]$. _____

4 Consider the equivalence relation on $\mathbb{Z}$ given by

$(x, y) \in R$ if and only if $(4 \mid x$ AND $4 \mid y)$ OR $(4 \nmid x$ AND $4 \nmid y)$

a) How many equivalence classes does this relation have? _____

b) What are they?