TOP HAT

The "join code" is 08724
To participate in the cell phone polling,
Text to (315) 636 -0905 the four digit code for each question followed by the response.

Quantum pioneer Max Born is credited with the "statistical interpretation of the wavefunction". Max Born's grandaughter is ….

Discovering the Subatomic World

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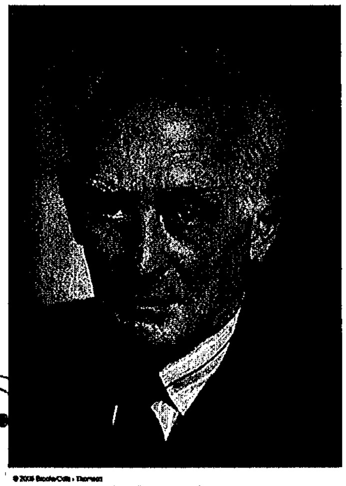
SUNY Distinguished Teaching Professor
SUNY Geneseo

NYS Master Teacher Program
March 2016



Minicourse Outline

1.) The electron

2.) The atomic nucleus

3.) and Beyond!

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Our Approach

- How do we know what we know?
- More experimental than theoretical
- Somewhat historical
- Engage in active learning!

Q: Rutherford scattering – infinite at zero degrees?

Small scattering angle - large impact parameter

$$N(\theta)=\frac{N_i nt}{16}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{Z_1^{\,2}Z_2^{\,2}}{r^2(K)^2\sin^4\left(\frac{\theta}{2}\right)}$$

Q: How did they know E_p in 1930s?

"The energy (hence speed) of the protons could be deduced by such means as determining what thickness of metal foil they could penetrate before being slowed down or by measuring how many ion pairs they created in a Geiger counter; such measurements were well-calibrated by this time."

http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.373.1481&rep=rep1&type=pdf

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Atomic mass vs. nuclear mass

tabulated.

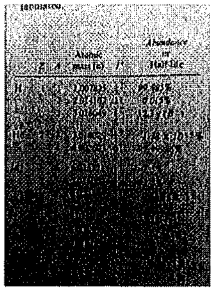
	Z	A	Atomic mass (u)	I^π	Abundance
H	1	1	1.007825	[illegible]	[redacted]
		2	2.014102	[illegible]	
		3	3.016049	[illegible]	
He	[illegible]	[illegible]	3.016029		
			[illegible]		

[redacted]

Kenr

$$^{3}He + {}^{2}H \rightarrow {}^{4}He + p$$

8



$$^{3}He + {}^{2}H \rightarrow {}^{4}He + p$$

$$mass_{initial} = M_{{}^{3}He} + M_{{}^{2}H}$$

$$= 3.016029u + 2.014102u$$

$$= 5.030131u$$

$$mass_{final} = M_{{}^{4}He} + M_{{}^{1}H} = 5.010428u$$

$$mass_{missing} = 0.019702u$$

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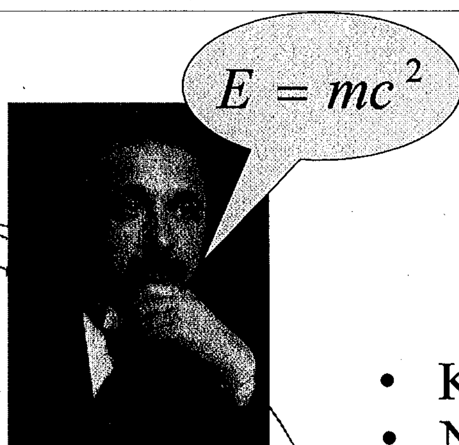
$$^{3}He + {}^{2}H \rightarrow {}^{4}He + p$$

$$mass_{missing} = 0.019702u = 18.35\ MeV / c^{2}$$



After this reaction, the proton and alpha have 18.35 MeV of "extra" energy

- Kinetic energy
- Nuclear Fusion

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https://upload.wikimedia.org/wikipedia/en/f/f7/Albert_Einstein_portrait.jpg

TOP HAT

Binding Energy of the nucleus

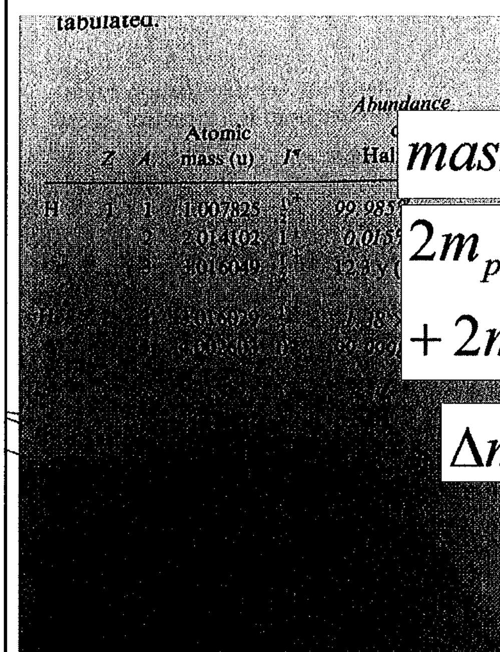
Ex.) Photodissociation of the deuteron

$$\gamma + {}^2H \rightarrow {}^1H + n$$

What's the energy of the gamma?

$$M_{{}^2H} = 1876.140\ MeV/c^2$$
$$M_{{}^1H} = 938.791\ MeV/c^2$$
$$M_n = 939.573\ MeV/c^2$$

The gamma ray must have an energy of at least *2.22 MeV*



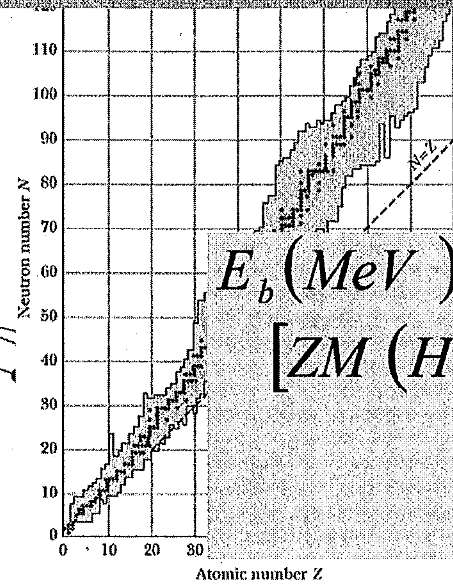
Similarly…

$$mass_{helium} = 4.002603u$$

$$2m_{proton} + 2m_{neutron} + 2m_{electron} = 4.03298u$$

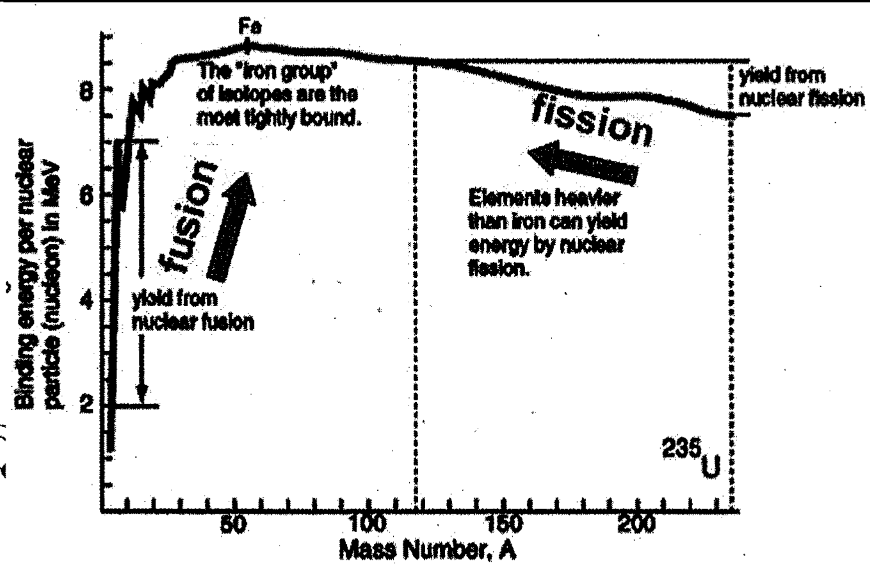
$$\Delta m = 28.30 MeV/c^2$$

Mass nucleus < mass of nucleons



Binding Energy of the nucleus

$$E_b(MeV) = [ZM(H) + Nm_N - M_A] \times 931.502 \frac{MeV}{u}$$



http://h2physics.org/?cat=82

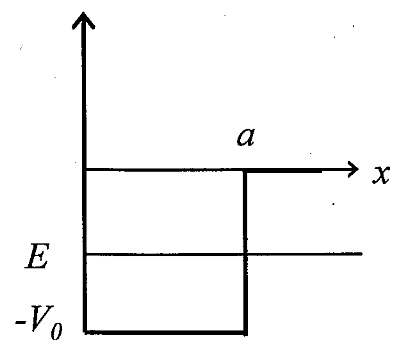
A Model for the Deuteron

Consider a neutron in the potential energy {V(x)} field provided by the strong force attraction of the proton.

$$V(x)=\begin{cases}+\infty, & x<0\\ -V_0, & 0<x<a\\ 0, & a<x\end{cases}$$

$$V(x)=\begin{cases}+\infty, & x<0\\ -V_0, & 0<x<a\\ 0, & a<x\end{cases}$$

$$-V_0<E<0$$



Time Independent Schrödinger Equation

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+V(x)\psi(x)=E\psi(x)$$

IN REGION $0 < x < a$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + (-V_0)\psi = E\psi \, , \quad E = -|E|$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = (V_0 - |E|)\psi$$

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}(V_0 - |E|)\psi$$

$$= -k^2\psi \, , \quad k = \sqrt{\frac{2m}{\hbar^2}(V_0 - |E|)}$$

GENERAL SOLN

$$\psi = A\sin(kx) + B\cos(kx)$$

B.C. AT $x = 0$ $\psi = 0$ $\therefore$ $B = 0$

$$\boxed{\psi = A\sin(kx)} \quad 0 < x < a$$

IN REGION $x > a$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + 0 = -|E|\psi$$

$$\frac{d^2\psi}{dx^2} = +\frac{2m}{\hbar^2}|E|\psi$$

$$= +\beta^2\psi, \quad \beta = \sqrt{\frac{2m}{\hbar^2}|E|}$$

GENERAL SOLN

$$\psi = C e^{\beta x} + D e^{-\beta x}$$

AT $x = \infty$ $\psi = 0$ $\therefore$ $C = 0$

$$\boxed{\psi = D e^{-\beta x}} \quad x > a$$

AT $x = a$ B.C. $\psi_>(a) = \psi_<(a)$

BC $\frac{d\psi_>(a)}{dx} = \frac{d\psi_<(a)}{dx}$

SO

① $A \sin(ka) = D e^{-\beta a}$

② $A k \cos(ka) = -\beta D e^{-\beta a}$

DIVIDE ② BY ①

$$k \cot(ka) = -\beta$$

$$\boxed{\cot(ka) = -\beta/k}$$

HOW DO WE SOLVE THIS?

$$\beta = \sqrt{\frac{2m}{\hbar^2}|E|}$$

$$k = \sqrt{\frac{2m}{\hbar^2}(V_0 - |E|)}$$

$\cot(ka) = -\beta/k$

TRANSCENDENTAL EQN

NUMERICAL METHOD.

$\longrightarrow$

FOR $V_0 = 40$ MeV

$a = 1.89$ fm

$E = -2.22$ MeV !

Solution

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- For other values of V_0 and a we can obtain more than one energy that solves TISE
- *Quantized Energy Levels*
- Shell model for nucleons (protons and neutrons separate)

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Inside the nucleus there are protons and neutrons

TOP HAT

What's inside the proton?

How do we know?

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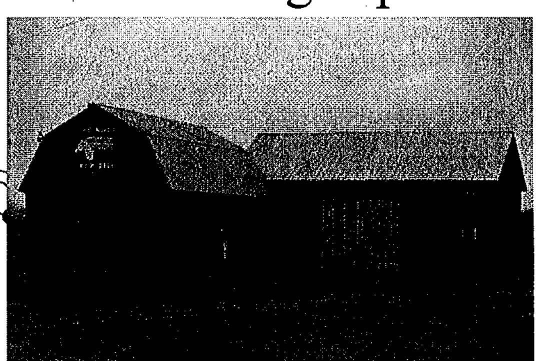
"Deep inelastic scattering experiments"

First – Define "Cross section" for scattering experiments.



"Broadside of a barn door"

http://backroadstraveller.blogspot.com/2013/11/all-kinds-of-barns.html

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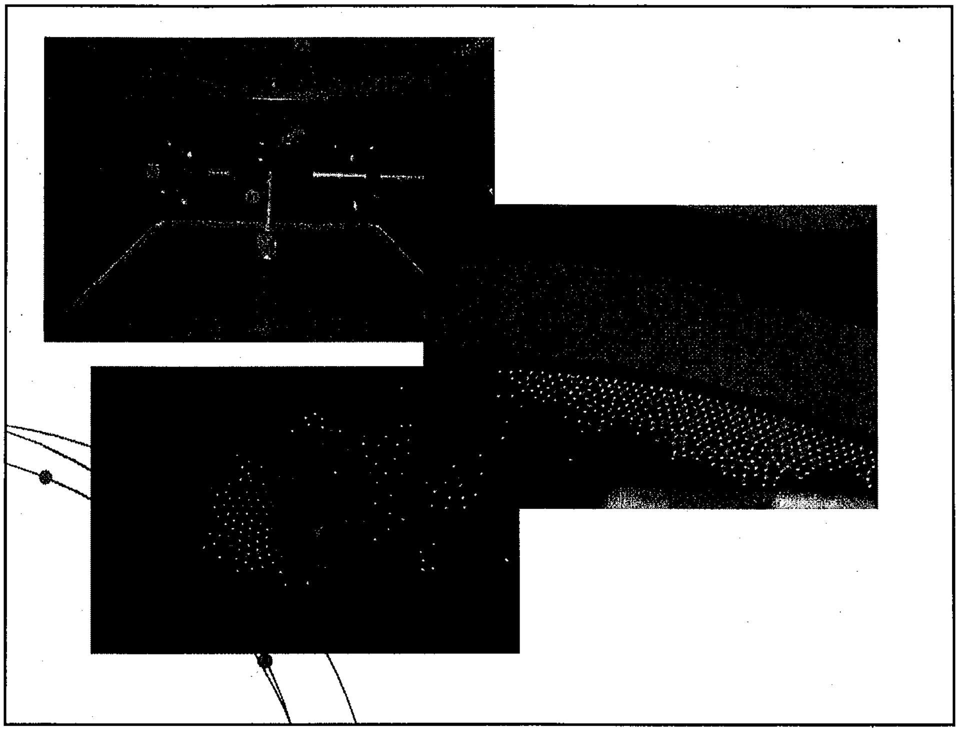
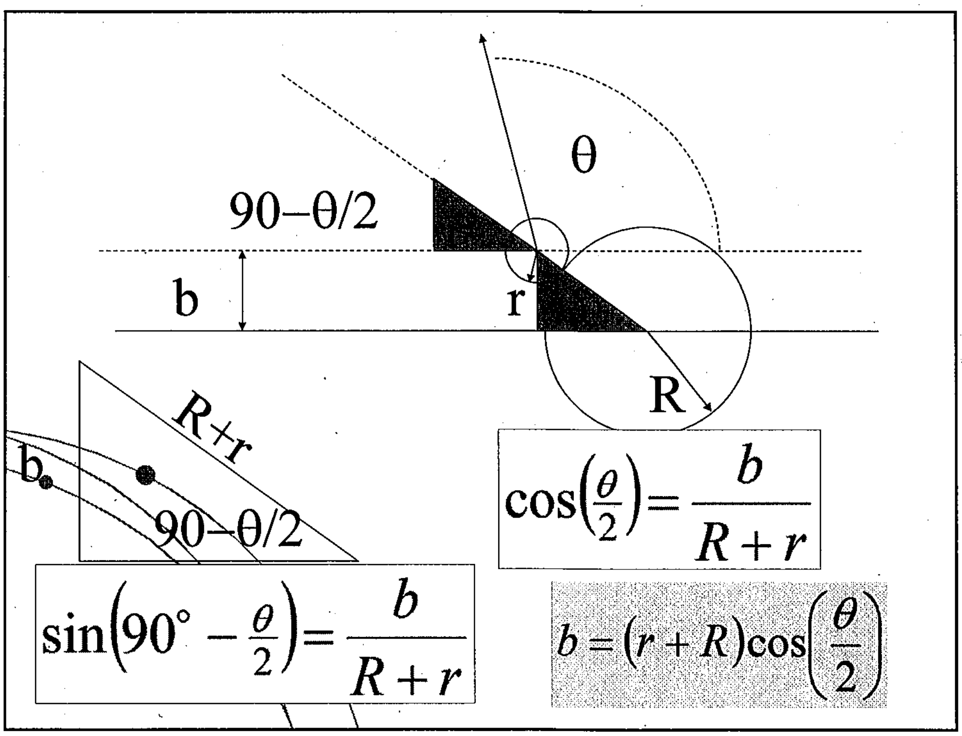
Recall Rutherford Scattering

$$N(\theta)=\frac{N_i n t}{16}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{Z_1^{\;2}Z_2^{\;2}}{r^2(K)^2\sin^4\left(\frac{\theta}{2}\right)}$$

A Macroscopic Analogy





$$\cos\left(\frac{\theta}{2}\right) = \frac{b}{R+r}$$

$$\sin\left(90^\circ - \frac{\theta}{2}\right) = \frac{b}{R+r}$$

$$b = (r+R)\cos\left(\frac{\theta}{2}\right)$$

$$dN = A \cdot db$$

The # of incident projectiles that come between b and $b+db$

The # of projectiles per width of bb beam $=N_{bb}/x$

What's the expression for db ?

$$b = (r+R)\cos\left(\frac{\theta}{2}\right)$$

$$db = \frac{1}{2}(r+R)\sin\left(\frac{\theta}{2}\right)d\theta$$

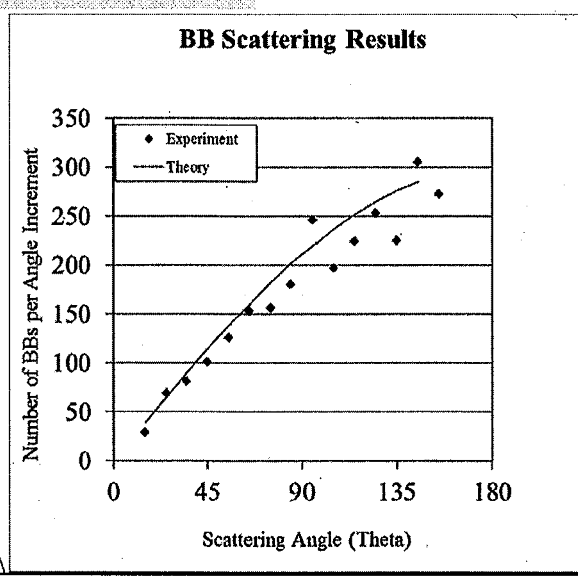
$$dN = \frac{N_{bb}}{x}\frac{1}{2}(r+R)\sin\left(\frac{\theta}{2}\right)d\theta$$

The # of incident projectiles that scatter between θ and $\theta + d\theta$

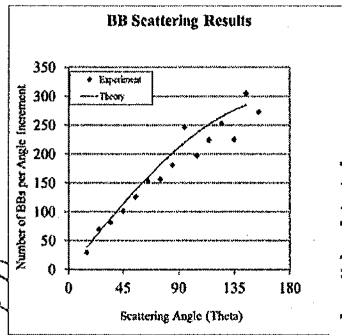
The angular size of the detector

$$dN = \frac{N_{bb}}{x}\frac{1}{2}(r+R)\sin\left(\frac{\theta}{2}\right)d\theta$$

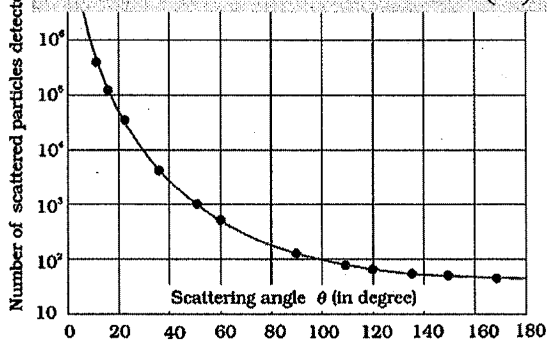
Vary R to match the data.



$$dN = \frac{N_{bb}}{x}\frac{1}{2}(r+R)\sin\left(\frac{\theta}{2}\right)d\theta = \frac{N_{bb}}{x}\frac{d\sigma}{d\theta}d\theta$$



$$N(\theta) = \frac{N_i nt}{16}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{Z_1^2 Z_2^2}{r^2 (K)^2 \sin^4\left(\frac{\theta}{2}\right)}$$



$$dN = \frac{N_{bb}}{x}\frac{1}{2}(r+R)\sin\left(\frac{\theta}{2}\right)d\theta$$

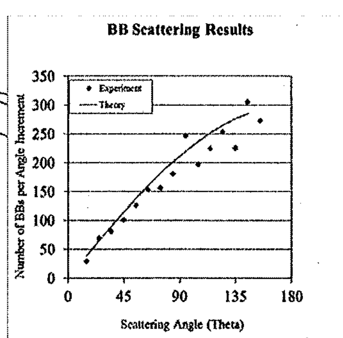
Number of particles scattered into this detector

Number of projectiles fired at the target

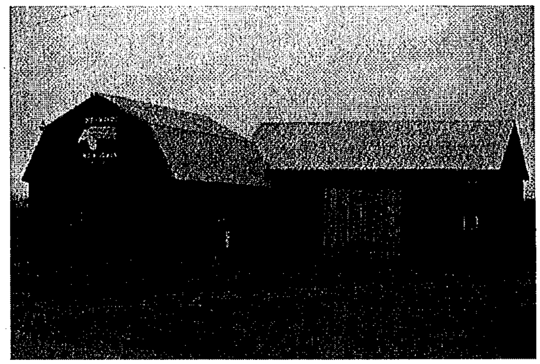
$$dN = \frac{N_{bb}}{x}\left(\frac{d\sigma}{d\theta}\right)d\theta$$

"Cross section" for the target

Size of the detector



"Cross section" is proportional to the probability that the scattering will occur.



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"Deep inelastic scattering experiments"

- High energy electrons (at SLAC) fired at nuclei in stationary targets.

http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/scatele.html#c1

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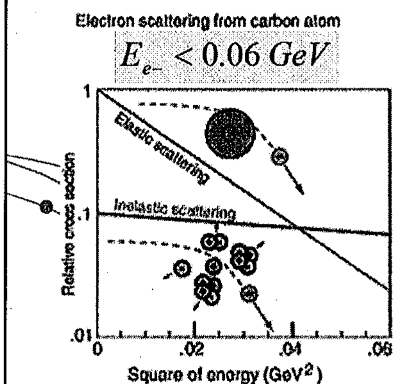
"Deep inelastic scattering experiments"

- σ for elastic scattering (nucleus remains intact) decreases dramatically

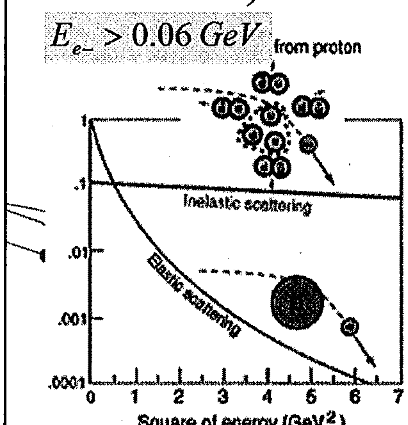


- σ for inelastic scattering (nucleus breaks apart) pretty constant
- Suggests e⁻ scatters off of bits (p's and n's) that don't break up (for these E_e)

http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/scatele.html#c1 Discovering the Subatomic World 33

At Higher Energies....

- σ for elastic scattering (proton remains intact) decreases dramatically



- σ for inelastic scattering (proton breaks apart) pretty constant
- Suggests e⁻ scatters off of bits (quarks) that don't break up

http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/scatele.html#c1 Discovering the Subatomic World 34

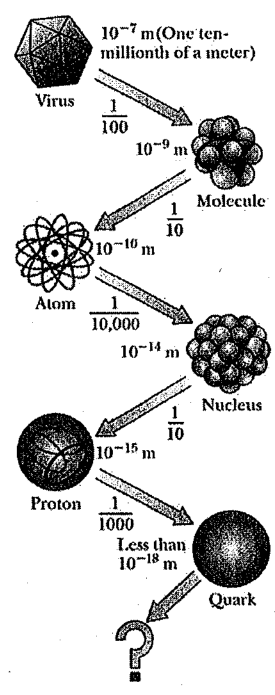
A Field Trip to the Subatomic Particle Zoo-

The Fundamental Particles and Interactions Chart – basic particle physics

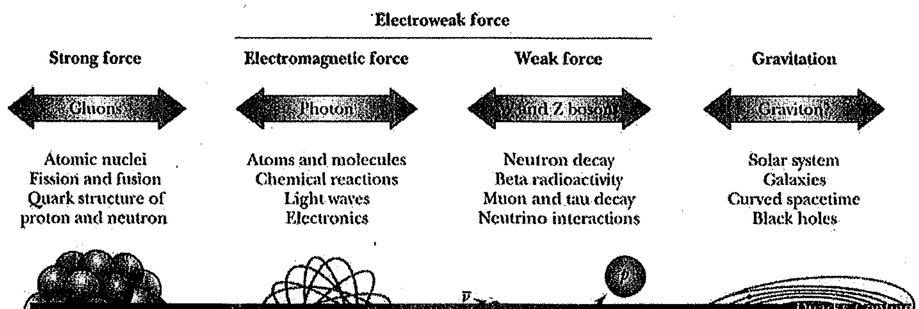
http://www.cpepweb.org/cpep_sm_large.html

Atomism –
Greek Atomos-

"Indivisible"



The Fundamental Interactions



Properties of the Interactions

Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
[illegible]	Mass – Energy	Flavor	Electric Charge	Color Charge
[illegible]	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
[illegible]	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
[illegible]	10^{-41}	0.8	1	25
[illegible]	10^{-41}	10^{-4}	1	60

http://www.cpepweb.org/cpep_sm_large.html

Matter and Antimatter

Fermions and Bosons

all stable matter in the universe appears to be composed, at some level, of constituent fermions.

Leptons spin = 1/2

Flavor	Mass GeV/c²	Electric charge
ν_L lightest neutrino*	$(0-2)\times10^{-9}$	0
e electron	0.000511	−1
ν_M middle neutrino*	$(0.009-2)\times10^{-9}$	0
μ muon	0.106	−1
ν_H heaviest neutrino*	$(0.05-2)\times10^{-9}$	0
τ tau	1.777	−1

Leptons -

-pointlike (“elementary”)
-do not interact via strong interaction

Lepton Conservation

- Three families of Leptons,
- *The number of leptons from each family is the same both before and after a reaction.*
- $L_e = +1$ for the electron and the electron neutrino; $L_e = -1$ for their antiparticles; and $L_e = 0$ for all other particles.
- L_μ for the muon and its neutrino and L_τ *for the tau and its neutrino similarly.*

$$\mu \rightarrow e + \nu_\mu + \overline{\nu}_e$$

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c²	Electric charge
u up	0.002	2/3
d down	0.005	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	173	2/3
b bottom	4.2	−1/3

Quarks -

Hadrons -

-composite particles
-made up of quarks
-DO interact via Strong interaction

Mesons & Baryons
The two types of Hadrons

Table 14.6 Quark Composition of Selected Hadrons

Particle	Quark Composition
Mesons	
π^+	$u\bar{d}$
π^-	$\bar{u}d$
K^+	$u\bar{s}$
K^0	$d\bar{s}$
D^+	$c\bar{d}$
D^0	$c\bar{u}$
Baryons	
p	uud
n	udd
Λ	uds
Σ^+	uus
Σ^0	uds
Ξ^0	uss
Ξ^-	dss
Ω^-	sss
Λ_c^+	udc

Mesons

Quark-antiquark

Bosons

All unstable
Not abundant

Baryons

3 Quark-combinations

Fermions

Proton –only stable baryon?
Others decay to protons

Table [illegible] The Hadrons

Particle Name	Symbol	Anti-particle	Mass (MeV/c^2)	Mean Lifetime (s)	Main Decay Modes	Spin	Baryon Number B	Strangeness Number S	Charm Number C
Mesons									
Pion	π^+	π^-	140	2.6×10^{-8}	$\mu^+\nu_\mu$	0	0	0	0
	π^0	Self	135	8.4×10^{-17}	2γ	0	0	0	0
Kaon	K^+	K^-	494	1.2×10^{-8}	$\mu^+\nu_\mu, \pi^+\pi^0$	0	0	1	0
	K_S^0	$\overline{K}_S^0$	498	8.9×10^{-11}	$\pi^+\pi^-, 2\pi^0$	0	0	1	0
	K_L^0	$\overline{K}_L^0$	498	5.2×10^{-8}	$\pi^\pm e^\mp \nu_e, 3\pi^0, \pi^\pm\mu^\mp\nu_\mu, \pi^+\pi^-\pi^0$	0	0	1	0
Eta	η^0	Self	547	5×10^{-19}	$2\gamma, 3\pi^0, \pi^+\pi^-\pi^0$	0	0	0	0
Charmed D's	D^+	D^-	1868	1.0×10^{-12}	$e^+, K^\pm, K^0, \overline{K}^0$ + anything	0	0	0	1
	D^0	$\overline{D}^0$	1864	4.1×10^{-13}	Same as D^+	0	0	0	1
	D_s^+	$\overline{D}_s^-$	1969	4.9×10^{-13}	Various	0	0	1	1
Bottom B's	B^+	B^-	5279	1.7×10^{-12}	Various	0	0	0	0
	B^0	$\overline{B}^0$	5279	1.5×10^{-12}	Various	0	0	0	0
J/Psi	J/ψ	Self	3097	10^{-20}	Various	0	0	0	0
Upsilon	$\Upsilon(1S)$	Self	9460	10^{-20}	Various	0	0	0	0
Baryons									
Proton	p	$\overline{p}$	938.3	Stable (?)		$\frac{1}{2}$	1	0	0
Neutron	n	$\overline{n}$	939.6	886	$pe^-\overline{\nu}_e$	$\frac{1}{2}$	1	0	0
Lambda	Λ^0	$\overline{\Lambda}^0$	1116	2.6×10^{-10}	$p\pi^-, n\pi^0$	$\frac{1}{2}$	1	−1	0
Sigmas	Σ^+	$\overline{\Sigma}^-$	1189	8.0×10^{-11}	$p\pi^0, n\pi^+$	$\frac{1}{2}$	1	−1	0
	Σ^0	$\overline{\Sigma}^0$	1193	7.4×10^{-20}	$\Lambda^0\gamma$	$\frac{1}{2}$	1	−1	0
	Σ^-	$\overline{\Sigma}^+$	1197	1.5×10^{-10}	$n\pi^-$	$\frac{1}{2}$	1	−1	0
Xi	Ξ^0	$\overline{\Xi}^0$	1315	2.9×10^{-10}	$\Lambda^0\pi^0$	$\frac{1}{2}$	1	−2	0
	Ξ^-	$\overline{\Xi}^+$	1321	1.6×10^{-10}	$\Lambda^0\pi^-$	$\frac{1}{2}$	1	−2	0
Omega	Ω^-	Ω^+	1672	0.82×10^{-10}	$\Lambda^0 K^-, \Xi^0\pi^-$	$\frac{3}{2}$	1	−3	0
Charmed lambda	Λ_c^+	$\overline{\Lambda}_c^-$	2285	2.0×10^{-13}	Various	$\frac{1}{2}$	1	0	1

Quantum Chromodynamics

Consider the proton - *uud*

- Quarks are fermions (spin = ½),
- So how can two up quarks be in the same state? (Pauli Exclusion)

additional quantum number (called Color) for the state of the quark

- Color – "charge" of strong nuclear force (Red, Green, Blue, anti-Red, etc.)

All hadrons are colorless!

Baryon Conservation

- For low-energy nuclear reactions, the number of nucleons is always conserved.
- Define *baryon number* - $B = +1$ for baryons and -1 for antibaryons, and 0 for all other particles.
- *Conservation of baryon number*

Strangeness Conservation

- *Strangeness number S* = some integer – property of particles.
- *Conservation of strangeness number*

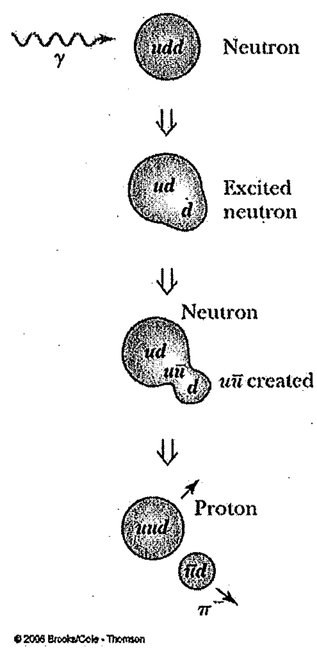
$$\gamma + n \rightarrow p + \pi^-$$

Quark Confinement-

Strong force *increases* with distance

Extra energy produces more quarks – forming Hadrons

No "free" quarks



http://www.cpepweb.org/cpep_sm_large.html