LAB 1

MATH 233 - LINEAR ALGEBRA I
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1   Matlab as a calculator

When you open up Matlab you will encounter a prompt that looks like

```
>>
```

After entering a command at the prompt you press the ENTER key to execute the command.
    You can begin typing expressions as you would in a calculator. For example, to compute

$$-1 - \frac{3}{7}(0.333 + \sqrt{7.6}) - \ln(0.83) + 0.44^2 - \sin(3.3)\cos(-0.11) + \tan(-28)$$

you input

```
>> -1-3/7*(0.333+sqrt(7.6))-ln(0.83)+0.44^2 - sin(3.3)*cos(-0.11)+tan(-28)
```

To evaluate the exponential function $f(x) = e^x$ we use $\exp(x)$. For example, to compute $e^{-9.33}$ we type

```
>> exp(-9.33)
```

Matlab also has some built-in mathematical constants, such as (of course) π. For example, to compute $3\pi^2$ we type:

```
>> 3*pi^2
```

You can assign variables to expressions. For example,

```
>> x=11*sqrt(7)
```

and then access the variable by simply calling it from the prompt. For example,

```
>> x^2-2*x+1
```

2   Matrices

Matrices in Matlab are entered one row at a time (row-by-row), with the entries in a row separated by space or a comma (,), and rows are separated by a semi-colon (;). For example, to enter the matrix

$$\mathbf{M} = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 7 & 11 & -5 \end{bmatrix}$$

we type

```
>> M=[1 -2 1 0;0 2 -8 8;-4 7 11 -5]
```

Using commas to separate entries in a row, we could have also entered **M** by typing

```
>> M=[1,-2,1,0;0,2,-8,8;-4,7,11,-5]
```

If we add a semi-colon at the end of a Matlab input, then the output is suppressed. Try adding a semi-colon when defining **M**:

```
>> M=[1 -2 1 0;0 2 -3 8;-4 7 11 -5];
```

To see a matrix you have created, or any other stored variable, you simply type its name

```
>> M
```

Or the variable x previously defined:

```
>> x
```

Matlab holds in memory about 30 of your most recent input commands. You can scroll through your previous input commands by pressing the UP-ARROW at the prompt. You can edit a previous input by simply typing your changes and then re-executing it by pressing ENTER. To see how this can be useful, let's enter the following matrix but make an error in our input command:

$$\mathbf{N} = \begin{bmatrix} 2 & -3 \\ 4 & 7 \\ 11 & -3 \\ 0 & 1 \end{bmatrix}$$

Type

```
>> N=[2 -3;4 7;11 -3;0 1 3]
```

Do you notice what the input error is? Matlab gives an error because the last row contains three entries, while all the other rows contain only two. Press the UP-ARROW repeatedly to view our previous input commands and then press the DOWN-ARROW to go back to the last input command used to input **N**. Directly edit the input and delete the 3 in the last row and then press ENTER.

An important task done frequently is to change/reassign the entries of a matrix or to use the entries of a matrix to define a new matrix. To extract or access the (i, j) entry of **M** we simply type M(i,j). For example, to extract the $(3, 2)$ entry of **M** we type

```
>> M( 3 , 2 )
```

We can also extract groups of entries of **M**. To extract rows 2 and 3 and columns 3 and 4 we type

```
>> M( 2 : 3 , 3 : 4 )
```

To extract a whole row, say the 3rd row, we type

```
>> M( 3 , : )
```

or to extract a whole column, say the 4th column, we type

```
>> M( : , 4)
```

We could extract the entries of a matrix and use them to define a new matrix. For example, to create a new matrix from the matrix **M** using rows 2 to 3 and columns 2 to 4 we type:

```
>> R = M(2 : 3, 2 : 4)
```

Another way to do this is:

```
>> R = M(2 : , 2 : )
```

Notice that whenever Matlab encounters a colon : it extracts to the end of rows/columns.

3 Solving systems of linear equations with `rref`

Consider the linear system

$$2x_1 + 4x_2 + 6x_3 = 8$$
$$x_1 + 2x_2 + 4x_3 = 8$$
$$3x_1 + 6x_2 + 9x_3 = 12$$

Define the coefficient matrix of the system and call it **A**.

```
>> A=[2 4 6;1 2 4;3 6 9]
```

Now define the image vector **b**:

```
>> b=[8; 8; 12]
```

To create the augmented matrix $\mathbf{M} = [\mathbf{A} \ \mathbf{b}]$ we can simply enter

```
>> M=[A b]
```

or

```
>> M=[A,b]
```

Equivalently, we could have entered **M** directly one entry at a time:

```
>> M=[2 4 6 8;1 2 4 8;3 6 9 12]
```

To compute the RREF of the augmented matrix **M** we use Matlab's built-in function `rref`:

```
>> rref(M)

ans =
       1     2     0     -8
       0     0     1      4
       0     0     0      0
```

Therefore the system is consistent. The number of leading 1's is $r = 2$ and therefore there is $d = 3 - 2 = 1$ free parameter. The second row of rref($\mathbf{M}$) gives that $x_3 = 4$. We then set $x_2 = t$ to be the free parameter in the solution set. From the first equation of rref($\mathbf{M}$) we obtain that $x_1 + 2x_2 = -8$ and therefore $x_1 = -8 - 2t$. Thus, the solution to the linear system is

$$x_1 = -8 - 2t$$
$$x_2 = t$$
$$x_3 = 4$$

Because there are free parameters in the solution set (in this case just $d = 1$), there are an infinite number of solutions. If for instance we set $t = 1$ then we obtain the solution $x_1 = -10$, $x_2 = 1$, and $x_3 = 4$.

As another example, consider the linear system corresponding to the augmented matrix

$$\mathbf{M} = \begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

The matrix $\mathbf{M}$ is a 3×6 matrix. The last column of $\mathbf{M}$ is the image vector $\mathbf{b}$, namely,

$$\mathbf{b} = \begin{bmatrix} -5 \\ 9 \\ 15 \end{bmatrix}$$

and the coefficient matrix $\mathbf{A}$ is

$$\mathbf{A} = \begin{bmatrix} 0 & 3 & -6 & 6 & 4 \\ 3 & -7 & 8 & -5 & 8 \\ 3 & -9 & 12 & -9 & 6 \end{bmatrix}.$$

The coefficient matrix $\mathbf{A}$ is a 3×5 matrix, that is, $\mathbf{A} \in \mathbb{R}^{3 \times 5}$. Thus, $\mathbf{M}$ is the augmented matrix of a linear system consisting of $n = 5$ unknowns in $m = 3$ equations. To solve the linear system corresponding to $\mathbf{M}$, we compute the RREF of $\mathbf{M}$:

```
>> rref(M)
```

From the RREF of $\mathbf{M}$ we see that there are $r = 3$ leading 1's and therefore there are $d = n - r = 2$ free parameters in the solution set of the linear system.

Matlab has a very useful built-in help system. To get the help file for any command, such as rref, you simply type

```
>> help rref
```

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4   Vector operations and Span

In Matlab, algebraic operations can be performed on vectors just as with regular scalars. In Matlab, define the vectors

$$\mathbf{v}_1 = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 4 \\ -2 \\ 4 \end{bmatrix}$$

using the command

```
>> v1=[-3; 1; 2];
>> v2=[2; 0; -3];
>> v3=[4; -2; 4];
```

We can compute a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, say $3\mathbf{v}_1 - 4\mathbf{v}_2 - 7\mathbf{v}_3$, by simply typing

```
>> 3*v1 -4*v2 -7*v3

ans =
    -45
     17
    -10
```

Now let

$$\mathbf{b} = \begin{bmatrix} 12 \\ -4 \\ -3 \end{bmatrix}.$$

Is $\mathbf{b} \in \mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? As we learned in class, to answer this question we could solve the linear system having coefficient matrix $\mathbf{A} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3]$ and augmented matrix $\mathbf{M} = [\mathbf{A} \ \mathbf{b}]$. We can define $\mathbf{M}$ in Matlab by directly inputting each entry of $\mathbf{M}$ but we have already defined $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ in Matlab, and thus we can simply define $\mathbf{M}$ by first defining $\mathbf{b}$

```
>> b=[12;-4;-3];
```

and then defining $\mathbf{M}$ using

```
>> M=[v1 v2 v3 b]

M =
    -3     2     4     12
     1     0    -2     -4
     2    -3     4     -3
```

Note that we can define $\mathbf{A}$ also using

```
>> A=[v1 v2 v3];
```

As we learned in class, $\mathbf{b} \in \mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ if the linear system corresponding to $\mathbf{M} = [\mathbf{A} \ \mathbf{b}]$ is consistent, and this can be verified by computing the RREF of $\mathbf{M}$:

```
>> rref(M)

ans =
        1    0    0   -2
        0    1    0    1
        0    0    1    1
```

Therefore, the linear system is consistent and thus $\mathbf{b} \in \mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Moreover, we see from the result of `rref` that

$$\mathbf{b} = -2\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3.$$

HAND-IN PROBLEMS

Instructions: Use Matlab to solve the following problems. Write your answers in a separate sheet of paper and hand it in at the end of the lab time. You may work in groups but hand in your own lab paper. Make sure to write your name and clearly identify what question you are answering.

Problem 1: What is
$$\frac{\sqrt{2}\left(\sqrt{3+\pi}\right)^3}{e^2 - 1}$$
up to four decimal places?

Problem 2: Consider the linear system

$$x_1 + 2x_2 - 2x_3 + x_4 = -3$$
$$2x_1 + x_2 - x_3 + 2x_4 = 0$$
$$x_1 - 4x_2 + 4x_3 + x_4 = 9$$

  (i) What is the RREF of the augmented matrix $\mathbf{M}$ of the linear system?
 (ii) Is the linear system consistent?
(iii) How many leading 1's or pivots are in the RREF of $\mathbf{M}$?
 (iv) How many free parameters are there in the solution set of the linear system?
  (v) How many solutions are there in the solution set?
 (vi) Write the solution set in parametric form and in vector form.
(vii) What is one solution of the linear system?

Problem 3: Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} -2 \\ -1 \\ -3/2 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} 3 \\ 2 \\ -1/2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}.$$

  (i) What is
$$-\mathbf{v}_1 - 2\mathbf{v}_2 + 4\mathbf{v}_3?$$
 (ii) Does there exist scalars x_1, x_2, x_3 such that
$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{b}?$$
      If yes, what are they? Are they unique? Why?
(iii) Is $\mathbf{b} \in \mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Why?