Problem Set 1 — Limits

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Math 221 05

Complete By Thursday, September 7
Grade By Tuesday, September 12

Purpose

This problem set mainly develops your ability to use limit laws and algebra to find limits. Secondarily, it also consolidates the ideas of tangent lines and areas that motivate calculus.

Background

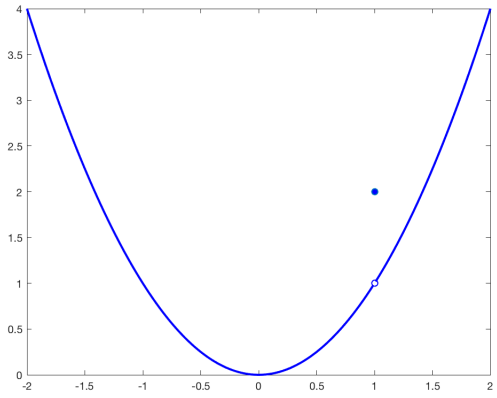
This exercise is based on material in sections 2.1 through 2.3 of our textbook. We will cover this material in class between August 30 and September 6.

Section 2.2 introduces the idea of a limit, while 2.3 develops the limit laws and algebraic techniques that can be used to formally find limits. Section 2.1 provides an overview of the main ideas behind calculus, notably tangent lines and areas.

Activity

Solve the following problems:

Problem 1. Based on the following graph, estimate the limit of the function (1) as x approaches 0, and (2) as x approaches 1.



Problem 2. Use limit laws and algebra to find the following limits.

$$\lim_{x\to 3} \frac{2x}{x+1} \tag{1}$$

$$\lim_{x\to -1} \frac{x^2+2x+1}{x+1} \tag{2}$$

$$\lim_{x\to \sqrt{3}} 0 \tag{3}$$

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} \tag{4}$$

Problem 3. Consider the function $f(x) = x^2 + x$.

Part 1. Find the equations of two secant lines to this function, one between the points $(1, f(1))$ and $(1.2, f(1.2))$, and the other between the points $(1, f(1))$ and $(1.1, f(1.1))$. You may use a calculator or similar technology in doing this.

Part 2. Use algebra and limit laws to find

$$\lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$$

Part 3. What does the limit in Part 2 have to do with the secant lines in Part 1?

Problem 4. Consider the line $y = x$.

Part 1. Use rectangles to estimate the area below this line between $x = 2$ and $x = 3$. Calculate two estimates, one dividing the region into 2 equal-width rectangles, and the other dividing it into 4 equal-width rectangles; in all cases let the rectangles be below the line. You may use a calculator or similar technology in this part.

Part 2. Use area formulas from geometry to find the exact area below the line between $x = 2$ and $x = 3$. You may use a calculator or similar if you want.

Part 3. Does it seem reasonable that the area estimated using rectangles has the exact area as a limit as the number of rectangles increases? Why or why not?

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.