

Problem Set 10 — Integration by Substitution

Prof. Doug Baldwin

Math 221 05

Complete By Sunday, November 26

Grade By Wednesday, November 29

Purpose

This problem set reinforces your ability to compute definite integrals and antiderivatives, particularly using the substitution technique.

Background

This problem set mainly draws on material from section 5.5 of our textbook. We covered that material in classes beginning November 15.

One exercise is based on material about area from section 5.3, which we discussed in class on November 13.

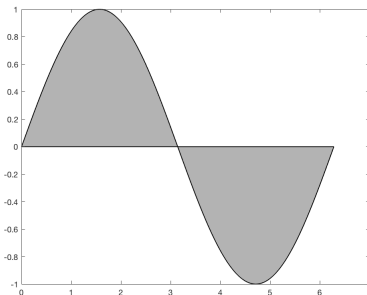
Activity

Solve the following problems.

Problem 1. Prove the following extension to the power rule for antiderivatives: For all constants $n \neq -1$ and k ,

$$\int (x + k)^n dx = \frac{1}{n + 1} (x + k)^{n+1}$$

Problem 2. Find the area between the x axis and one cycle of a sine wave, i.e., the shaded area in this plot:



Problem 3. Find the following indefinite integrals:

1. $\int 3x^2(1 + x^3)dx$
2. $\int (7x - 11)^4 dx$ (OpenStax *Calculus, Volume 1*, Problem 274 in Section 5.5.)
3. $\int t^2 \cos^2(t^3) \sin(t^3) dt$ (OpenStax *Calculus, Volume 1*, Problem 280 in Section 5.5.)
4. $\int y^2 \sqrt{1 + y} dy$

Problem 4. Evaluate the following definite integrals:

1. $\int_0^1 x \sqrt{1 - x^2} dx$ (OpenStax *Calculus, Volume 1*, Problem 292 in Section 5.5.)
2. $\int_0^\pi \cos^2(2\theta) \sin(2\theta) d\theta$ (Based on OpenStax *Calculus, Volume 1*, Problem 306 in Section 5.5.)

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the “Grade By” date above.