Problem Set 11 — Area and Volume via Integration

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Math 221 05

Complete By Sunday, December 3
Grade By Wednesday, December 6

Purpose

This problem set develops your ability to solve various kinds of problem using integration. In doing so, it also reinforces your ability to evaluate definite integrals.

Background

This problem set mainly draws on material from sections 6.1 and 6.2 of our textbook. We covered that material in classes beginning November 20.

Activity

Solve the following problems.

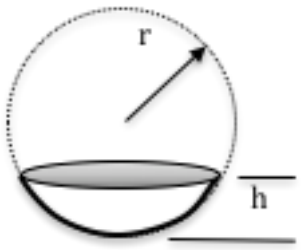
Problem 1. (OpenStax *Calculus, Volume 1*, Problem 2 in Section 6.1.)
Find the area between the graphs of $y = x^2$ and $y = 3x + 4$. See the textbook for a picture of the graphs and the area between them.

Problem 2. (OpenStax *Calculus, Volume 1*, Problem 16 in Section 6.1.)
Find the area between the graphs of $y = \sin x$ and $y = \cos x$ over the interval $-\pi \leq x \leq \pi$.

Problem 3. (From OpenStax *Calculus, Volume 1*, Problem 68 in Section 6.2.)
Sketch a solid whose base is a circle of radius a and whose cross sections perpendicular to that base are squares. Use the slicing method to find the volume of this solid.

Problem 4. (From OpenStax *Calculus, Volume 1*, Problem 78 in Section 6.2.)
Sketch the region bounded by the curves $y = \sqrt{x}$, $x = 0$, $x = 4$, and $y = 0$. Then find the volume of the solid produced by rotating that region around the y axis.

Problem 5. A "cap" of a sphere is a piece sliced off the sphere's side by a plane. The quantities that determine the size of the cap are the sphere's radius, r, and the height h of the cap (i.e., the distance from the slicing plane to the edge of the sphere):



Part 1. Derive a formula for the volume of a cap of a sphere in terms of r and h. You may assume $h \leq r$.

Part 2. Suppose you have a hemispherical bowl of radius 5 inches. If you're pouring water into the bowl at a rate of 1 cubic inch per second, how fast is the depth of water in the bowl increasing when the bowl contains 100 cubic inches of water? Note: you probably solved Part 1 by first expressing the volume of a cap as a certain integral, and then evaluating that integral to get the final formula. You can exceed my expectations for at least this question by using that initial integral from Part 1, but not the formula it evaluates to, to solve Part 2.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.