Problem Set 2 — Limits

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Math 221 05

Complete By Sunday, September 17
Grade By Wednesday, September 20

Purpose

This exercise reinforces your ability to find limits through algebraic manipulation and limit laws. It also develops your understanding of more advanced limit concepts, notably one-sided limits, infinite limits, and the formal definition of limit.

Background

This problem set is based on material in sections 2.2 through 2.5 of our textbook. We discussed, or will discuss, these ideas in classes between September 7 and 14.

Activity

Solve the following problems:

Problem 1. Use algebra to find

$$\lim_{x \to \frac{\pi}{2}} (\cos x \tan x)$$

You will probably have to use your intuition about a certain limit at some point in solving this problem, but you can rewrite the original limit algebraically before you need that intuition.

Problem 2. Find

$$\lim_{x \to 1^-} \frac{x^2 - 1}{|x - 1|}$$

and

$$\lim_{x \to 1^+} \frac{x^2 - 1}{|x - 1|}$$

Use muPad to plot $y = \frac{x^2-1}{|x-1|}$ near $x = 1$ and verify that what you see is consistent with the limit(s) you calculated.

Does $\lim_{x\to 1} \frac{x^2-1}{|x-1|}$ exist? If so, what is it? If not, why not?

Problem 3. Find

$$\lim_{x\to 3} \frac{1}{x^2 - 2x - 3}$$

How many vertical asymptotes does $\frac{1}{x^2-2x-3}$ have? Where are they?

Use muPad to graph $\frac{1}{x^2-2x-3}$ and check that the graph is consistent with your answers to the previous two questions.

Problem 4. (From OpenStax, *Calculus Volume 1*, section 2.5.)

Prove that

$$\lim_{x\to 2} \frac{2x^2 - 3x - 2}{x - 2} = 5$$

Problem 5. One of the basic limit laws says that

$$\lim_{x\to a} c = c$$

for any constants a and c. Prove this law.

Problem 6. Newton's law of gravitation says that the gravitational force pulling bodies of masses m_1 and m_2 (measured in kilograms) together is

$$F = G\frac{m_1 m_2}{r^2}$$

where r is the distance between the bodies in meters and G is a constant approximately equal to 6.67×10^{-11} $\mathrm{Nm}^2/\mathrm{kg}^2$. The force, F, is measured in newtons.

Conveniently, for purposes of this law symmetrical spherical masses can be treated as if all their mass is concentrated in a point at their center. This means, for example, that the force of gravity at the surface of a star or planet can be calculated using the star or planet's radius as r and its total mass as m_1.

Imagine that you, with some constant mass, are standing on the surface of a star[1] when it starts collapsing. The star collapses in such a way that its mass remains constant, but gets packed into an ever smaller sphere. Throughout this collapse, you keep standing on the shrinking surface of the star. Calculate your weight (i.e., the force of the star's gravity on you) in the limit as the star's radius approaches 0. To what extent do you think you can rely on this mathematical result as a model of physical reality? (For brownie points, but maybe not actual grade points, what is a star called if it collapses to the point that its radius approaches 0?)

[1] OK, there's a lot about standing on the surface of a star that needs a pretty active imagination...

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the “Grade By” date above.