Problem Set 3 — Derivatives

Prof. Doug Baldwin

Math 221 05

Complete By Sunday, September 24
Grade By Wednesday, September 27

Purpose

This exercise reinforces your ability to find derivatives from the definition of "derivative" and from some differentiation rules. It also further consolidates your understanding of the formal definition of limits.

Background

This problem set is mostly based on material in sections 3.1 through 3.3 of our textbook. We discussed, or will discuss, these ideas in classes between September 18 and 22. Problem 1 is based on material in section 2.5, discussed in class on September 13 and 14.

Activity

Solve the following problems:

Problem 1. Our textbook uses the squeeze and composite function theorems to prove that $\lim_{x \to a} \sin x = \sin a$ for any constant a. As a consequence, you know that

$$\lim_{x \to \frac{\pi}{2}} \sin x = 1 \tag{1}$$

Illustrate how this result is consistent with the formal definition of limit by drawing a graph of $\sin x$ near $x = \frac{\pi}{2}$ with appropriate intervals marked for a general ϵ and the corresponding δ. Be sure you are able to explain how your graph illustrates Equation 1.

Problem 2. Use the limit definition of the derivative at a point to find the derivatives of the following functions at the given points:

1. $f(x) = 3x - 2$ at $x = 1$.

2. $f(x) = x^2 + 2x - 2$ at $x = -1$.

Problem 3. Use the limit definition of the derivative as a function to find functions that are the derivatives of the following:

1. $f(x) = x^2 + 2$

2. $f(x) = \frac{1}{x-1}$

Problem 4. Use differentiation laws to evaluate the following derivatives:

1. $\frac{dy}{dx}$ given that $y = 3x^2 - 5x + 7$

2. $\frac{dr}{dt}$ given that $r = \sqrt{t^3}$

3. $\frac{d^2y}{dx^2}$ given that $y = \frac{1}{2}x^2 - 3x$

4. $\frac{d^2s}{da^2}$ given that $s = \frac{3}{a}$

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.