Problem Set 4 — Derivatives

Prof. Doug Baldwin

Math 221 05

Complete By Sunday, October 1
Grade By Wednesday, October 4

Purpose

This exercise reinforces your understanding of differentiation rules. It particularly (but not exclusively) focuses on the product and quotient rules, and on derivatives of trigonometric functions. This problem set also introduces you to finding antiderivatives, and to using muPad to find derivatives and antiderivatives.

Background

This problem set is based on material in sections 3.3 and 3.5 of our textbook. We covered (or will cover) those sections in class between September 21 and September 28.

Our September 22 lecture notes on "Differentiation Rules" demonstrate how to find derivatives and antiderivatives with muPad, something that isn't covered in the textbook.

Activity

Solve the following problems.

Problem 1. Find the following derivatives. *After* finding each derivative by hand, use muPad to find the derivative, and check that your answer and muPad's agree. This may require some algebra, as muPad doesn't always simplify things the way you probably do.

1. $\frac{dz}{dx}$ if $z = 2\sin x - 3\tan x$
2. $h'(a)$ if $h(a) = (a^2 + 5)\cos a$
3. $\frac{ds}{dt}$ if $s = (t+1)\frac{t^2-1}{t^2+2}$

Problem 2. Find the general antiderivative of the following functions. *After* finding each antiderivative by hand, use muPad to find the antiderivative, and check that your answer and muPad's agree. As in Problem 1, this may require some algebra or interpretation on your part.

1. $f(t) = 2t^3 - 6t + 1$
2. $g(x) = \frac{3}{x^2}$
3. $r(\Theta) = 5 + \cos\Theta$

Problem 3. Part 1. Suppose $f(x)$ is a differentiable function. Use the product rule to show that the derivative of $(f(x))^2$ is $2f(x)f'(x)$.

Part 2. Use the result from Part 1 to find $g'(x)$ if $g(x) = \sin^2 x$.

Problem 4. While it is not one of the standard differentiation rules, there could be a "reciprocal rule": If $f(x)$ is a differentiable function and $g(x) = \frac{1}{f(x)}$, then

$$g'(x) = \frac{-f'(x)}{(f(x))^2}$$

Prove this rule. You may use either the limit definition of the derivative or differentiation rules discussed in class.

Problem 5. You may use a calculator on both parts of this question.

Part 1. Imagine an elliptical oil slick that at some moment in time is 3 meters long and 2 meters wide. At this instant, it's length is increasing at a rate of 2 centimeters per minute, and its width at a rate of 1 centimeter per minute. How fast is the oil slick's area increasing?

Helpful information: The area of an ellipse is given by the equation $A = \pi ab$, where A is area, and a and b are the lengths of the semi-major and semi-minor axes respectively. What does that mean? The long axis of an ellipse is called its *major axis*; the short axis is called the *minor axis*. Half of these axes, i.e., the lines from the center of the ellipse to its edge in the longest and shortest dimensions, are the *semi-major* and *semi-minor* axes.

Part 2. If the oil slick is a constant 1 millimeter thick, how fast (in units of volume per minute, e.g., cubic centimeters per minute) is oil entering the slick at the moment described in Part 1?

More helpful information: The volume of such an oil slick is simply its thickness times its area.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.