Problem Set 5 — The Chain Rule

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Math 221 05

Complete By Sunday, October 15
Grade By Wednesday, October 18

Purpose

This exercise's main purpose is to reinforce your understanding of the chain rule (including its use in implicit differentiation). As a secondary purpose, this exercise also gives you a little more practice working with derivatives of trigonometric functions.

Background

This problem set is mainly based on material in sections 3.6 and 3.8 of our textbook. We covered (or will cover) those sections in class between September 29 and October 4.

Trigonometric derivatives are covered in section 3.5 of our textbook, and our September 28 lecture.

Activity

Solve the following problems.

Problem 1. Derive the formula for the derivative of $\sec x$. (In other words, show how the equation $\frac{d}{dx}(\sec x) = \sec x \tan x$ follows from the definition of $\sec x$ as $\frac{1}{\cos x}$ and previous differentiation rules or the limit definition of the derivative.)

Problem 2. Find $\frac{dy}{dx}$ given the following definitions of y. These problems all come from OpenStax *Calculus, Volume 1*:

1. $y = (3x-2)^6$ (Section 3.6, exercise 220).
2. $y = x^2 \cos^4 x$ (Section 3.6, exercise 234).
3. $y = \sqrt{6 + \sec(\pi x^2)}$ (Section 3.6, exercise 236).

Problem 3. Find $\frac{dy}{dx}$ given the following definitions of y in terms of u and u in terms of x. These problems also all come from OpenStax *Calculus, Volume 1*:

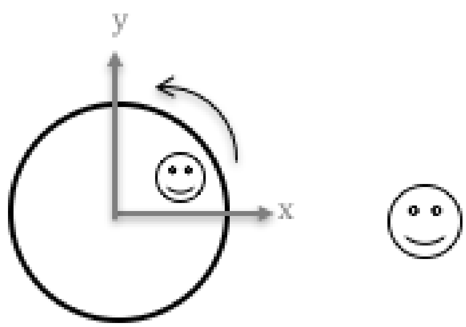
1. $y = 6u^3, \quad u = 7x - 4$ (Section 3.6, exercise 215).

2. $y = \sin u, \quad u = 5x - 1$ (Section 3.6, exercise 216).

Problem 4. Exercise 238 in Section 3.6 of OpenStax *Calculus, Volume 1*: Let $y = (f(x))^3$ and suppose that $f'(1) = 4$ and $\frac{dy}{dx} = 10$ when $x = 1$. Find $f(1)$.

Problem 5. Exercise 300 in Section 3.8 of OpenStax *Calculus, Volume 1*: Use implicit differentiation to find $\frac{dy}{dx}$ if $x^2 - y^2 = 4$.

Problem 6. Imagine a child riding on an amusement park carousel. There is a coordinate system conveniently superimposed on the carousel, so that positions relative to the center of the carousel can be given as (x, y) pairs. In this system, the child's x coordinate t minutes after the ride starts is given by $x = 10\cos(8\pi t)$ and the child's y coordinate by $y = 10\sin(8\pi t)$. The child's parent is standing at point $(20, 0)$. Here is a diagram of the situation:



How fast is the distance between the child and their parent changing? Give your answer as a function of t.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.