Problem Set 6 — Related Rates

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Math 221 05

Complete By Sunday, October 22
Grade By Wednesday, October 25

Purpose

This exercise's main purpose is to reinforce your ability to set up and solve related rates problems. It also provides a little bit of practice with linearization and miscellaneous calculus.

Background

This problem set is mainly based on material in section 4.1 (related rates) of our textbook. We covered that material in class between October 6 and 12.

Linearization is section 4.2 of the textbook, which we covered in our October 13 lecture.

Activity

Solve the following problems.

Problem 1. (OpenStax *Calculus, Volume 1*, section 4.1, exercise 16)

The side of a cube increases at a rate of $\frac{1}{2}$ m/sec. Find the rate at which the volume of the cube increases when the side of the cube is 4 m.

Problem 2. (OpenStax *Calculus, Volume 1*, section 4.2, exercise 52)

Find the linear approximation to $f(x) = \tan x$ near $x = \frac{\pi}{4}$.

Problem 3. (Inspired by OpenStax *Calculus, Volume 1*, section 4.1, exercises 10 and 11. Both strike me as a little unusual as related rates problems, but they are good exercises in calculus- and geometry-based problem-solving.)

A 6-ft tall person walks away from a 10-ft lamppost at a constant rate of 3 ft/sec. See the picture in our textbook for a visual presentation.

Part A. What is the rate at which the tip of the person's shadow moves away from the *person* when the person is 10 ft away from the pole?

Part B. What is the rate at which the tip of the person's shadow moves away from the *pole* when the person is 10 ft away from the pole?

Problem 4. (OpenStax *Calculus, Volume 1*, section 4.1, exercise 7)

Two airplanes are flying in the air at the same height: airplane A is flying east at 250 mi/hr and airplane B is flying north at 300 mi/hr. If they are both heading to the same airport, located 30 miles east of airplane A and 40 miles north of airplane B, at what rate is the distance between the airplanes changing?

Problem 5. (You may use calculators on this problem, but if you do be sure you can show me what you did and explain why during grading.)

Dr. Whowhatwhenwherewhyandhow, intrepid space-time traveller, is fleeing in a stolen spaceship from his enemies the Doombas[1]. The Doctor is accelerating away from the Doomba space station in such a manner that his speed relative to the space station t seconds after leaving is $20t$ meters/second.

Part A. How far will the Doctor be from the space station after t seconds? Assume that at $t = 0$,the moment he started fleeing, his speed was 0. (Hint: remember that speed is the derivative of distance.)

Part B. Unbeknownst to the Doctor, the Doombas have attached a bright light to his spaceship in order to track it through their telescopes. In fact, "bright light" is an understatement: the "radiant intensity"[2] of the light 1 meter away from it is 10^4 W/m^2. Radiant intensity decreases with the square of distance, so when the Doctor is r meters from the Doomba space station, his intensity as seen there will be $\frac{10^4}{r^2}$ W/m^2; his only hope of escape is therefore to get far enough away fast enough that the tracking light fades to invisibility before the Doombas find it. How fast is the intensity of the light as seen at the space station changing when the Doctor is 1 kilometer away?

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.

[1] a variety of robotic vacuum cleaner that has gone berserk and now seeks to destroy all life in the universe.

[2] A measure that more or less corresponds to how bright the light looks to a camera or eye observing it; by way of reference the average radiant intensity of the sun at Earth's surface is around 174 W/m^2.