Problem Set 7 — Graphs and Derivatives

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Math 221 05

Complete By Sunday, October 29
Grade By Wednesday, November 1

Purpose

Broadly speaking, this exercise reinforces your ability to use derivatives as estimators or approximators of a function's graph. In particular, it exercises derivatives as tools for finding extreme values, for estimating the rough shape of a graph, and for finding linear approximations to a function. This problem set also helps you understand the Mean Value Theorem and begins to develop your ability to find limits as x approaches ∞.

Background

This problem set is based on material in sections 4.2 through 4.6 of our textbook. We covered that material in class between October 13 and 23.

Activity

Solve the following problems.

Problem 1. (Based on OpenStax *Calculus, Volume 1*, section 4.3, exercise 120)
Use muPad or another computer algebra system to find the absolute and local maxima and minima of the function

$$y = (x - x^2)^2$$

over the interval $[-1, 1]$. Note: you need to find exact numeric values for the x and y coordinates of the maxima and minima, not just estimate them from a graph.

Problem 2. (Based on OpenStax *Calculus, Volume 1*, section 4.5, exercise 232)
Consider the function

$$f(x) = x + \sin(2x)$$

Using muPad or another computer algebra system, and considering the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$, find

1. Sub-intervals (if any) where $f(x)$ is increasing
2. Sub-intervals (if any) where $f(x)$ is decreasing
3. The local (if any) and absolute minima of $f(x)$
4. The local (if any) and absolute maxima of $f(x)$
5. Sub-intervals (if any) where $f(x)$ is concave up
6. Sub-intervals (if any) where $f(x)$ is concave down
7. Inflection points (if any) of $f(x)$.

Use muPad (or your other CAS) to graph $f(x)$ over the interval, and be prepared to point out on the graph each of the points or sub-intervals you found. But note that I want exact numeric values for the endpoints of intervals and locations and values of extrema, not just estimates from the graph.

Problem 3. (OpenStax *Calculus, Volume 1*, section 4.6, exercise 262)
Evaluate the limit

$$\lim_{x\to\infty} \frac{2x-5}{4x}$$

Problem 4. (Based on OpenStax *Calculus, Volume 1*, section 4.3, exercise 90)
One of the rules you may have learned in earlier courses (although I don't remember anyone teaching it in my pre-calculus courses) is that the minimum or maximum of a quadratic function

$$y = ax^2 + bx + c$$

occurs at

$$x = \frac{-b}{2a}$$

Use what you now know from calculus to derive this rule.

Problem 5. (Based on OpenStax *Calculus, Volume 1*, section 4.4, exercise 190)
At 10:17 a.m., you are traveling 55 mph when you pass a police car that is stopped on the freeway. You pass a second stopped police car at 10:53 a.m., when you are also traveling 55 mph. The second police car is located 39 mi from the first one. If the speed limit is 60 mph, can the police cite you for speeding? Why or why not?

Problem 6. The equation

$$xy^2 - 2yx^2 = -1$$

has as one of many solutions $x = -\frac{1}{2}$, $y = 1$ (i.e., the point $(-\frac{1}{2}, 1)$ lies on the graph of the equation). *Without* actually solving the equation, estimate a value of y that will satisfy the equation when $x = -0.49$.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.