Problem Set 9 — Riemann Sums and Definite Integrals

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Math 221 05

Complete By Tuesday, November 14
Grade By Friday, November 17

Purpose

This problem set mainly reinforces your ability to work with definite integrals as Riemann sums, although it also reinforces some material from earlier in this course.

Background

This problem set mainly draws on material from sections 5.1 through 5.3 of our textbook. We covered, or will cover, that material in classes between November 1 and 8.

Activity

Solve the following problems.

Problem 1. (OpenStax *Calculus, Volume 1*, Problem 150 in Section 5.3.)
Use the Fundamental Theorem of Calculus, Part 1, to evaluate

$$\frac{d}{dx}\int_3^x \sqrt{9-y^2}dy$$

Problem 2. (OpenStax *Calculus, Volume 1*, Problem 154 in Section 5.3.)
Use the Fundamental Theorem of Calculus, Part 1, to evaluate

$$\frac{d}{dx}\int_0^{\sin x} \sqrt{1-t^2}dt$$

Problem 3. (OpenStax *Calculus, Volume 1*, Problem 10 in Section 5.1.)
Use summation properties and formulas to find

$$\sum_{j=11}^{20}(j^2-10j)$$

Problem 4. Part 1. Find $\int_1^2 x dx$ by evaluating the limit of a Riemann sum. Check your answer against what you get using area formulas from geometry.

Part 2. Find $\int_0^1 x^2 dx$ by evaluating the limit of a Riemann sum.

Part 3. Using your answers to Parts 1 and 2, and the textbook's discovery that $\int_0^2 x^2 dx = \frac{8}{3}$, find

$$\int_1^2 3x^2 - 2x dx$$

without taking further limits of Riemann sums, and without using the Fundamental Theorem of Calculus.

Part 4. Confirm your answer to Part 3 by recalculating the integral using the Fundamental Theorem of Calculus (Part 2).

(Note: you will particularly need to show or explain your work in Parts 3 and 4 in order for it to be clear that you used different methods in each part.)

Problem 5. Suppose a function $g(x)$ is defined as

$$g(x) = \sqrt{f(x)}$$

and that

$$f'(x) = 2x - 4$$

Part 1. Find a formula for $f(x)$.

Part 2. Your solution to Part 1 should involve a constant of integration. If $g'(4) = 1$, what is the value of that constant?

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.