Math 221 03
Prof. Doug Baldwin

Problem Set 2 — More Limits

Complete by **Wednesday, February 13**
Grade by **Monday, February 18**

Purpose

This problem set reinforces some relatively advanced limit concepts. It also gives you a chance to start using Mathematica. In particular, by the time you finish this problem set you should be able to ...

- Use limit laws and/or algebra to evaluate one-sided limits
- Use one-sided limits to determine whether two-sided limits do or don't exist
- Use limit laws and infinite limit theorems to find infinite limits
- Recognize when and why a function is or is not continuous
- Use the formal definition of limit to prove things about limits
- Use Mathematica to plot functions.

Background

This problem set is based on material in sections 2.2 through 2.5 of our textbook. We will discuss this material in class between February 6 and 11. Our Canvas "modules" page can show you exactly which ideas we discuss on which days.

We talked about how to plot with Mathematica in class on January 30.

Activity

Solve the following problems:

Question 1. Find

$$\lim_{x \to 1^-} \frac{x^2 - 1}{|x - 1|}$$

and

$$\lim_{x \to 1^+} \frac{x^2 - 1}{|x - 1|}$$

Use Mathematica to plot $y = \frac{x^2-1}{|x-1|}$ near $x = 1$ and verify that what you see is consistent with the limit(s) you calculated. Note that "absolute value" in Mathematica is the "Abs" function.

Does $\lim_{x \to 1} \frac{x^2-1}{|x-1|}$ exist? If so, what is it? If not, why not?

Question 2. Consider the function $f(x)$ defined piecewise by

$$f(x) = \begin{cases} x^3 + 1 & \text{if } x < 0 \\ 1 - x^2 & \text{if } x \geq 0 \end{cases}$$

Part A. Find $\lim_{x \to 0^-} f(x)$.

Part B. Find $\lim_{x \to 0^+} f(x)$.

Part C. Does $\lim_{x \to 0} f(x)$ exist? If so, what is its value? If not, why not?

Part D. Is $f(x)$ continuous at $x = 0$. Explain why or why not.

Question 3. (Based on exercise 154 in section 2.4 of Openstax *Calculus Volume 1.*)

Using the graph in exercise 154 (at the top of page 192 in our PDF of the book), say whether the function $f(x)$ is or is not continuous at each of the following x values:

1. $x = -2$
2. $x = -1$
3. $x = 0$
4. $x = 1$
5. $x = 2$

For each value at which the function is discontinuous, say what part of the definition of continuity is violated, and identify the discontinuity as a jump, infinite, or removable discontinuity.

Question 4. Newton's law of gravitation implies that the acceleration due to gravity at the surface of a star of mass m (measured in kilograms) is

$$a = G\frac{m}{r^2}$$

where r is the radius of the star in meters and G is a constant approximately equal to 6.67×10^{-11} Nm^2/kg^2.

Imagine that a spaceship (evidently a very hardy one) is skimming the surface of a star that starts collapsing. The star collapses in such a way that its mass remains constant, but gets packed into an ever smaller radius, and throughout the collapse the spaceship keeps flying right along the surface of the star. Calculate the acceleration due to the star's gravity on the spaceship in the limit as the star's radius approaches 0.

To what extent do you think you can rely on this mathematical result as a model of physical reality? (For brownie points, but maybe not actual grade points, what is a star called if it collapses to the point that its radius approaches 0?)

Question 5. (Exercise 188 in section 2.5 of Openstax *Calculus Volume 1.*)

Use the formal definition of limit to prove that

$$\lim_{x \to 2}(5x + 8) = 18$$

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along. Be sure to also bring either a print-out of your Mathematica work, or a computer with your work loaded into Mathematica.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.