Let D be the set of odd natural numbers, and $f : \mathbb{N} \to D$ be defined by $f(n) = 2n - 1$. Then...

Theorem 1. *f is an injection.*

Proof. We assume that $f : \mathbb{N} \to D$ is defined by $f(n) = 2n-1$, and prove that f is an injection. We do this by showing that if a and b are two natural numbers such that $f(a) = f(b)$, then $a = b$. Thus we assume that $f(a) = f(b)$, i.e., that $2a - 1 = 2b - 1$. Now adding 1 to both sides of this equation and then dividing by 2 yields $a = b$, which is what we wanted to show. Thus, we have proven that if $f : \mathbb{N} \to D$ is defined by $f(n) = 2n - 1$, then f is an injection. □

Theorem 2. *f is a surjection.*

Proof. We assume that $f : \mathbb{N} \to D$ is defined by $f(n) = 2n - 1$, and prove that f is an surjection by showing that every element of the codomain has a preimage in the domain. Specifically, let y be an element of D, i.e., $y = 2a + 1$ for some integer a and $y \geq 1$. Since $y \geq 1$, we see that $2a+1 \geq 1$, or that $a \geq 0$. Furthermore the preimage of y is the n such that

$$2n - 1 = y = 2a + 1$$

Solving this equation for n we get

$$\begin{aligned} n &= \frac{2a+1+1}{2} \\ &= \frac{2a+2}{2} \\ &= a+1 \end{aligned}$$

Now since a is an integer greater than or equal to 0, n is an integer greater than or equal to 1, i.e., a natural number.

To check that the n thus found really is a preimage of y , calculate

$$\begin{aligned} f(n) &= 2n - 1 \\ &= 2(a+1) - 1 \\ &= 2a + 2 - 1 \\ &= 2a + 1 \\ &= y \end{aligned}$$

This completes the demonstration that if $f : \mathbb{N} \to D$ is defined by $f(n) = 2n - 1$, then f is an surjection. □

Theorem 3. *f is a bijection.*

Proof. By Theorem 1, f is an injection, and by Theorem 2, f is a surjection. Thus from the definition of bijection, f is a bijection. □