## Math 333

Hopefully this exercise will help you understand how most of the things we have learned are related. This expands on the Big Theorem we learned in class involving square matrices by considering non-square matrices.

Exercise: Read through the following statements. Determine which statements are equivalent, and group equivalent statements together. You should be able to explain why they are equivalent, but do not prove the equivalence (unless you want to.) Every statement is equivalent to at least one other statement. Let $A$ be an $m \times n$ matrix and let $T$ be the linear transformation given by $T(\mathbf{x})=A \mathbf{x}$.
(a) $A \mathbf{x}=\mathbf{b}$ has at least one solution for any $\mathbf{b}$.
(b) The reduced row echelon form of $A$ is the identity matrix.
(c) $T$ is one-to-one, but not necessarily onto.
(d) $A$ has a free column, and the rows of $A$ are linearly independent.
(e) All rows of $A$ are linearly independent, and all columns of $A$ are linearly independent.
(f) $A$ is invertible.
(g) $T$ is onto, but not necessarily one-to-one.
(h) $A \mathbf{x}=\mathbf{b}$ has at most one solution for any $\mathbf{b}$.
(i) $T$ is both one-to-one and onto.
(j) $A \mathbf{x}=\mathbf{0}$ has only the trivial solution.
(k) $A$ has $m$ pivot columns.
(l) $T$ is invertible.
(m) The null space of $A$ is trivial.
(n) $A \mathbf{x}=\mathbf{b}$ has exactly one solution for any $\mathbf{b}$.
(o) The columns of $A$ are linearly independent.
(p) $A \mathbf{x}=\mathbf{0}$ has only the trivial solution, and $m=n$.

