

Math 333 - Linear Algebra

Final Exam Review

Fall 2007

1 Sections Covered

The final exam will have two parts. The first part will be cumulative, covering material from the first two exams. The second part will cover the material we have learned since the last exam.

For Part 1, you should go over the review sheets and practice exams for the first two exams. This time around it will be mostly computational, but you are expected to know all definitions and theorems mentioned on those review sheets. The proofs will be emphasized significantly less on this part of the final exam.

Part 2 of the final exam will cover Sections 3.1-3.4, 4.1-4.4, and 5.1-5.2 of the textbook. You should read and understand all of these sections.

2 Definitions

You should know ALL definitions covered in the above mentioned sections, not just those listed below. However, you should pay special attention to the definitions of each of the following terms. DEFINITELY KNOW (i.e. MEMORIZE) THESE DEFINITIONS.... Also, if asked to give a definition of something, you can NOT give a theorem. For example, “an eigenvalue λ of a matrix A is any root of the characteristic polynomial $\det(A - \lambda I) = 0$ ” is NOT the definition; it is a theorem.

1. **Chapter 3:** Elementary row operations, Augmented matrix, Homogeneous and Inhomogeneous systems, Consistent system, Row echelon form, Reduced row echelon form.
2. **Chapter 5:** Diagonalizable matrix and linear transformation, eigenvalue/eigenvector of a matrix and linear transformation, eigenspace, characteristic polynomial, algebraic multiplicity, geometric multiplicity.

For Sections 5.1 and 5.2, many definitions are given in the book in terms of linear transformations, but in class the definitions were given in terms of matrices. If you are asked for the definition of something involving matrices, then I want you to write a definition IN TERMS OF MATRICES, and there should be NO MENTION of a linear transformation (and vice versa). For example, for the definition of “Diagonalizable matrix”, you should NOT say,

“The square matrix A is diagonalizable if the linear transformation L_A is diagonalizable.”

Full credit will only be given for an answer such as

“The square matrix A is diagonalizable if A is similar to a diagonal matrix,” or
“ $A \in M_{n \times n}$ is diagonalizable if $A = Q^{-1}DQ$ for some diagonal matrix D .”

This warning applies to ALL of the definitions from Sections 5.1 and 5.2. If you have any doubts or concerns about a definition, please feel free to ask before and/or during the final exam.

3 Theorems

You should know the statements and understand the usefulness of all theorems in the above mentioned sections. Pay special attention to each of the following theorems. You will NOT be asked to write the precise statements of most of these theorems, so don't attempt to memorize exactly what they say. Simply know how to use them. However, you may be asked to write the precise statement if it has a ** next to it.

1. **Chapter 3:** Corollaries 2 and 3 (pg. 158-9), Theorems 3.9, 3.10, 3.11, 3.14
2. **Chapter 4:** Theorems 4.2, 4.7, 4.8, Corollary to 4.7 (pg. 223). Also know how row operations affect determinants.
3. **Chapter 5:** Theorems 5.2**, 5.4** (stated in terms of matrices too), 5.5, 5.7**
4. **Combination of Theorems 5.1** and 5.9**:**

An $n \times n$ matrix A is diagonalizable \iff there is a basis of F^n consisting entirely of eigenvectors v_i of A ,
 \iff for every eigenvalue λ_i of A , algebraic multiplicity = geometric multiplicity,

5. You should know part of the “**Big Theorem**”. That is, know at least 6 things we have learned in this class that are equivalent to a matrix A being nonsingular.

4 Computations and other Exercises

You are responsible for knowing any homework exercise that has been assigned as well as similar computational exercises. However, you may choose to focus on the following types.

1. Solve a system of linear equations using the Gaussian elimination method.
2. Compute the inverse A^{-1} of a matrix A using the Gaussian elimination method. Also be able to determine the inverse of a linear transformation.
3. Be able to calculate the rank of a matrix and the determinant of a matrix via expansion along any row or column.
4. Find eigenvalues, eigenvectors, and eigenspaces for a matrix or a linear transformation, and determine algebraic and geometric multiplicities. REMEMBER that you can check your answers by using the DEFINITION $A\vec{v} = \lambda\vec{v}$.

5. Be able to determine if a matrix or linear transformation is diagonalizable, and be able to find the diagonal matrix D and invertible matrix Q such that $Q^{-1}AQ = D$.
6. Other Exercises: Sec. 3.2: (3, 8), Sec. 4.2: (25, 27), Sec. 4.3 (10, 12), Sec. 5.1 (3, 8(c)), Sec. 5.2 (2, 8)

5 Proofs

You should be familiar with the various proof techniques we learned in class so that you will be able to do proofs and justifications similar to the homework problems. For example, how do you show that one set is contained in another set, how do you show that two sets are equal, or how do you show something is unique? We went over several different methods, so review them carefully.

You should also be familiar with the proofs of each of the following theorems. Let V and W be vector spaces and $T : V \rightarrow W$ be a linear transformation.

1. **Theorem 4.2:** A 2×2 matrix A is invertible if and only if $\det(A) \neq 0$.
2. **Theorem 5.2:** The number λ is an eigenvalue of the matrix A if and only if $\det(A - \lambda I) = 0$.
3. The vector v is an eigenvector of the matrix A if and only if $v \in \text{null}(A - \lambda I)$ and $v \neq 0$.

6 Good Luck!

Good luck on the exam! A lot of the work we have done has been computational, so make sure you can do all of the computational exercises from the homework. Make sure you practice enough of them so that the calculations can be done quickly (and *correctly*). Also be able to apply appropriate theorems to computational exercises to minimize the amount of work you have to do.

The theoretical portion of the exam will probably be the more difficult aspect simply because the ideas are somewhat new to you. However, the proofs themselves are not that difficult if you understand the material. A lot of the theoretical portion is listed on this review sheet as well as the practice exam. Remember that the point of this course is really to help you become more comfortable with proofs because you should already know how to do most of the computations. With enough practice, you should be fine.

You have all done well this semester. Keep up the good work, and don't get stressed! Study thoroughly and you should be okay. It has been a great pleasure having each of you in class. Best of luck on the final, and have a fantastic winter break!
