Math 380: Low-Dimensional Topology

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Textbook: *Topology Now!*, by Robert Messer & Philip Straffin.

Course Info: We will cover topological equivalence, deformations, knots and links, surfaces, three-dimensional manifolds, and the fundamental group. Topics are subject to change depending on the progress of the class, and various topics may be skipped due to time constraints. **An accurate reading schedule will be posted on the website, and you should check it often.**

By the time you take this course, most of you should be fairly comfortable with mathematical proofs. Although this course only has multivariable calculus, elementary linear algebra, and mathematical proofs as prerequisites, students are strongly encouraged to take abstract algebra (Math 330) prior to this course or concurrently. It requires a certain level of mathematical sophistication. There will be a lot of new terminology you must learn, and we will be doing a significant number of proofs.

Please note that we will work on developing your independent reading skills in Mathematics and your ability to learn and use definitions and theorems. I certainly won't be able to cover in class all the material you will be required to learn. As a result, you will be expected to do a lot of reading. The reading assignments will be on topics to be discussed in the following lecture to enable you to ask focused questions in the class and to better understand the material. It is imperative that you keep up with the reading assignments. Be prepared to learn many new definitions and theorems. It may be useful for you to compile a list of all the new terms we learn, as well as a list of theorems. Some students find flashcards useful for doing this.

Topology is the study of spaces and sets and can be thought of as an extension of geometry. It is an investigation of both the local and the global structure of a space or set. There are several areas of topology, and most of them have some overlap. Some of these include General Topology (or Point-Set Topology), Geometric Topology, Algebraic Topology, Differential Topology, Low-Dimensional Topology, High-Dimensional Topology, and various others. The usual topology course offered at Geneseo is General Topology (Math 338), which has its foundation in set theory, and this course will be significantly different from it. This course is an introduction to some topics in low-dimensional, geometric topology, and algebraic topology, including knots and links, surfaces and other manifolds, and the fundamental group. The course will allow students to see the exciting geometric ideas of topology and cultivate the intuitive ideas of continuity, convergence, and connectedness. The motivation behind topology is that some geometric problems do not depend on the exact shape of an object but on the way the object is put together. For example, the square and the circle are geometrically different but topologically the same. They have many properties in common: they are both one-dimensional objects and both separate the plane into two parts. Similarly, a donut and a coffee cup are topologically the same even though they look completely different. Much of the study of topology comes from setting aside our preconceived notions of "shape" involving size, length, flat, straight, or curved, and realizing that a circle and a square are really the same thing.

Upon successful completion of this course, a student will be able to:

- Analyze and demonstrate examples of knots, links, surfaces, and manifolds,
- Understand, use, and visualize the concepts of homeomorphism, deformation, and isotopy, and
- Understand and use various topological invariants of knots, links, and manifolds.

Grading: There will be regular homework assignments, two midterm exams, and a final exam. Your overall grade will be determined as shown below. **Class participation** will be based on your willingness to **ASK and ANSWER questions** in class. Please do not hesitate to ask questions when you have them.

Homework, Class Participation	19%
Exam 1:	27%
Exam 2:	27%
Final Exam: Thursday, Dec. 17, 8:00-11:00 am	27%

In all written work, you must show your work neatly and legibly in order to receive credit. You should clearly show the process and reasoning you went through in order to solve the problem. The problems I work for you in class will provide good examples of how your homework and exam problems should be written up. All assessment will be based on your ability to communicate a correct solution and explain your reasoning. It is absolutely essential to write clearly and completely. It is your responsibility to write in a way that tells me that you understand the problem and its solution.

Homework: There will be regular homework assignments that will be posted on the course website or given in class. The due dates will be given along with the assignment. To receive full credit, homework sets must be handed in to me **on time** (either in class or in my office.) Turn in as much of the homework as you can by the deadline to receive partial credit. If you have a legitimate conflict, you must tell me ahead of time.

Each student is responsible for the final preparation of his or her own homework papers. While the method of solution for a particular problem may be similar to that of other students, each student's presentation of the solution should be in his or her own words. However, **you are strongly encouraged to discuss the homework and to work together on the problems with your classmates.** Please be careful that you are actively participating in the process: many students find that they can understand a problem while they are watching a classmate work through it and explain it, and then conclude that they understand the material well enough. This leads to an unpleasant surprise at test time, when students who "thought they understood" the material find they are unable to work the problems on their own. Please be careful that you are able to work all of the problems on your own before the exam time arrives, with no coaching from a friend. Please use whatever resources aid you in learning the material, including computer assistance, office hours, other students, professors, math books, etc.

Extra Help: I am willing to spend a few minutes in class answering questions about homework problems. However, if you have a lot of questions, I recommend taking advantage of my office hours. I'll say it again…USE MY OFFICE HOURS! My job is to help you -- come to office hours even if you have just a small question. Don't wait until you get too far behind. If my office hours are not convenient for you, make an appointment by sending an email or asking after class. Please come see me as soon as you feel lost -- it is important that I know how you are doing so I can adjust the level of the class if necessary. I WANT to help you, and I WANT everyone to do well.

Don't let yourself get too far behind! I would be happy to help you if you feel you need some assistance. Come and get your questions cleared up right away.

Please Note: *Any student with a disability requiring accommodations is encouraged to contact me after class or during office hours. All discussions will remain confidential.*