Knot Mosaic Homework

Problem 1. For each knot and link discussed in this problem, provide a knot mosaic where the crossing number, mosaic number, and tile number are all realized.

  (a) Why is the mosaic number of the unknot 2?
  (b) What is the tile number of the unknot? Explain.
  (c) Determine all knots and links that can be placed on a 2×2 mosaic. Explain.
  (d) How many crossings can be placed on a 3×3 mosaic?
  (e) Determine all knots and links that can be placed on a 3×3 mosaic. Explain.

Problem 2. For each knot and link discussed in this problem, provide a knot mosaic where the crossing number, mosaic number, and tile number are all realized.

  (a) What is the mosaic number of the trefoil knot? Explain.
  (b) What is the tile number of the trefoil knot? Explain.

Problem 3. For each knot and link discussed in this problem, provide a knot mosaic where the crossing number, mosaic number, and tile number are all realized.

  (a) How many crossings can be placed on a 4×4 mosaic?
  (b) Determine all knots and links that can be placed on a 4×4 mosaic that cannot be place on a smaller mosaic. Explain. (If unknotted, unlinked components overlap other components in the knot mosaic diagram, you will use more tiles than necessary. Feel free to avoid this.)
  (c) What is the mosaic number of the figure-8 knot? Explain.
  (d) What is the tile number of the figure-8 knot? Explain.

Problem 4. For each knot and link discussed in this problem (except 6_1), provide a knot mosaic where the crossing number, mosaic number, and tile number are all realized. For the 6_1 knot, provide a knot mosaic where the mosaic number and tile number are all realized.

  (a) Why do the knots 5_1, 5_2, 6_1, 6_2, and 7_4 have mosaic number 5? Explain.
  (b) What are the tile numbers of these knots? Explain.

Problem 5. Observe the bounds for the tile number of a knot with mosaic number n for $n = 4, 5, 6, 7$, and 8. Why do you think it is more difficult to determine the tile number of a knot whose tile number is more than 5?

Problem 6. Opinion: Do you think it is possible to find a knot mosaic for any knot where the crossing number and mosaic number of the knot are both realized? Crossing number and tile number? Mosaic number and tile number?