Brenna Decker

Understanding the Properties of Linear Functions and Relations

analyzing a linear equation in terms of slope and intercepts

A linear function is a function with a constant rate of change.

A linear function can be represented in multiple ways.

- A linear function is written in standard form when it is written as Ax+By=C where x and y are variables, and A, B, and C are all integers.
- The linear function is written in slope-intercept form when it is notated as y=mx+b, where m is the slope and b is the y-intercept.
- A linear function is written in point-slope form when it is written as y-b=m(x-a), where (a, b) is a point the line passes through, and m is the slope.

The effects of slope and intercepts on the linear function:

Slope-

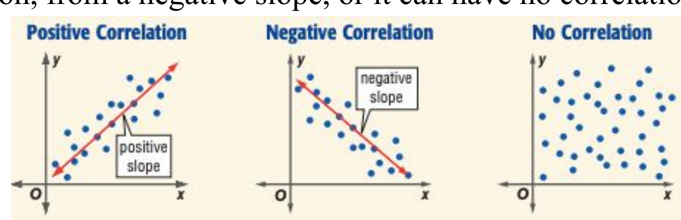
- Slope is considered the ratio between the change of output values of a function to the input values of a function. This also can be seen as the steepness of the line in a graphical setting.
- As the ratio of the change of output values to change of input values of the function becomes more positive, the slope will also become more positive. This will result in a positive, or upward, slope.
- As the ratio of the change of output values the change of input values of the function becomes more negative, the slope will also become more negative. This will result in a negative, or downward, slope.

y-intercept-

- The y-intercept is the y-coordinate of the point where the function meets the y-axis.
- As the y-intercept becomes more positive, the linear function is shifted up.
- As the y-intercept becomes more negative, the linear function is shifted down.
- In both situations, the slope of the line is preserved.

determining the linear function that best models a set of data

We can use linear functions to represent a set of data. We can do this by looking at a scatter plot, which is a graph in which two sets of data are plotted as ordered pairs. A set of data can either have a positive correlation, which we get from a positive slope, a negative correlation, from a negative slope, or it can have no correlation.



When the data points are correlated, we are able to draw a line of fit. A line of fit describes the trend of the data. From the line of fit, we will be able to find a linear equation that models the data. The line that models the data the closest is called the best-fit line.

There are two ways we can find the the best-fit line to a set of data. The first, and more accurate, is to input the set of data into a graphing calculator. Once the data is put into the calculator, we graph the scatter plot. Then, we can graph the line of best fit and get the slope and y-intercept from the calculator to form a linear equation.

The other method is graphically. We can graph our scatter plot, then estimate the best-fit by using a straight edge. Then, we can calculate the slope and y-intercept by estimating off of the graph.

Once we have our linear equation, we can interpolate and extrapolate data. Linear interpolation is using the linear equation to predict values that are inside the range of data. Linear extrapolation is used to predict values that are outside the range of data.

solving systems of linear equations and inequalities using a variety of techniques(algebraic, graphic, matrix)

A system of linear equations is a set of equations involving the same set of variables.

A solution to a system of linear equations is the set of numbers that can be plugged into the equation such that all the equations are simultaneously satisfied. A system of linear equations can have zero, one, or infinitely many solutions.

There are a few different ways to solve a system of linear equations. We can solve systems of linear equations with algebra, graphs, or matrices.

Solving Systems of Linear Equations Algebraically:

Elimination Method

In the elimination method, one equation is either added or subtracted from another in order to eliminate a variable. Sometimes, we have to multiply one equation by some constant in order to obtain matching coefficients on a variable.

Let's consider the following system of linear equations:

$$2x + 3y = 6$$
$$4x + 9y = 15$$

Now, we can multiply $2x + 3y = 6$ by 2, and we will get $4x + 6y = 12$. Then, we will subtract the equations.

$$4x + 6y = 12$$
$$-\ 4x + 9y = 15$$
$$\overline{\quad -3y = -3 \quad}$$

So, when we divide by -3 on each side, we get $y = 1$. Then, we plug 1 in for y to solve for x. We can use either equation to find x.

$$4x + 9(1) = 15$$
$$4x = 6$$
$$x = \tfrac{3}{2}$$

Substitution Method

To use the substitution method, we must solve one of the equations for a variable. Then, we plug that equation in to the other equation. Now, we only have one variable to solve for.

Consider the system of linear equations:

$$2x + 3y = 6$$

$$4x + 9y = 15$$

We can solve the top equation for x.

$$2x + 3y = 6$$
$$2x = 6 - 3y$$
$$x = 3 - \tfrac{3}{2}y$$

Then, we can plug this in to the other equation and solve for y.

$$4(3 - \tfrac{3}{2}y) + 9y = 15$$
$$12 - 6y + 9y = 15$$
$$12 - 3y = 15$$
$$-3y = -3$$
$$y = 1$$

Then, we can plug $y = 1$ into either equation to find $x = \frac{3}{2}$.

<u>Solving Systems of Linear Equations Graphically:</u>

To solve a system of linear equations graphically, we graph each line and see where they intersect. When solving a system of two variable linear equations graphically, there are three possible outcomes. There can either be zero, one or infinite solutions.

Case 1: Zero Solutions

A system of linear equations will have zero solutions when all of the linear equations are parallel to each other.

Case 2: One Solution

A system of linear equations will have one solution if the linear equations will intersect at one point.

Case 3: Infinite Solutions

Two linear equations that have infinite solutions are equivalent. This is because they intersect at every point along the line. In this case, the solution is every point on the lines.

<u>Solving Systems of Linear Equations with Matrices:</u>

Solving systems of linear equations using a matrix is very similar to using the elimination method. A <u>matrix</u> is a rectangular array of numbers enclosed in brackets. We must use <u>row operators,</u> or operations used on matrices, to get the <u>identity matrix</u>, which is the matrix in which when we multiply it by any other matrix, we get that same matrix.

Let us consider the system of linear equations:

$$2x + 3y = 6$$

$$4x + 9y = 15$$

Putting into matrix form, we have:

$$\begin{pmatrix} 2 & 3 & 6 \\ 4 & 9 & 15 \end{pmatrix}$$

We must get the two-by-two matrix produced by the coefficients of the variables into the form of the identity matrix. First, multiply the first row by two, and subtract it from the second row:

$$\begin{pmatrix} 2 & 3 & 6 \\ 0 & 3 & 3 \end{pmatrix}$$

Then, we subtract row 2 from row 1.

$$\begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & 3 \end{pmatrix}$$

Then, we multiply the top row by $\frac{1}{2}$, and the bottom row by $\frac{1}{3}$, and we get

$$\begin{pmatrix} 1 & 0 & \frac{3}{2} \\ 0 & 1 & 1 \end{pmatrix}$$

Thus, we have $x = \frac{3}{2}$ and $y = 1$.

A <u>system of linear inequalities</u> is a set of linear inequalities involving the same set of variables. The solution to a system of inequalities will be the set of all points that satisfy both inequalities.

Consider this set of linear inequalities:

$$y \geq 3x - 5$$
$$y \leq -x + 1$$

The solution set is $\{(x, y) | (y \geq 3x - 5) \cap (y \leq -x + 1)\}$.

<u>Solving Systems of Linear Inequalities Graphically:</u>

The best way to solve a system of linear inequalities is graphically. Let us consider the same set of linear inequalities. In order to graph each inequality, first we have to find where the boundary line is. We do this by using linear equations rather than inequalities, and getting the line into slope-intercept form. When we have the inequality to be $\leq$ *or* $\geq$, the boundary line will be a solid line, since we can include the points that are on the line. When we have the inequality to be $<$ *or* $>$, the boundary will be a dashed line, since we cannot include points on either line. Then, if the inequality is $>$ *or* $\geq$, we shade the region above the linear equation. If the inequality is $<$ *or* $\leq$, we shade the region below the boundary line. Wherever each shaded regions overlap, this will be the solution set that satisfies each linear inequality.

applying properties of linear equations and inequalities to model and solve a variety of real-world problems

Linear equations and inequalities can be used to model real-life situations. When we are given a real world problem, there are certain steps we can take to find a linear equation to fit it:

1. Identify known quantities.
2. Assign a variable to represent unknown quantity or quantities. Try to write one unknown in terms of the other if there are multiple.
3. Write the equation interpreting the words into mathematical operations.
    - It is important to note certain keywords for each operation. For example, more than indicates addition, less than indicates subtraction, product indicates multiplication and quotient indicates division.
4. Solve.

Linear inequalities can also be used to model real-life situations. We can take the same steps as above to find a linear inequality to find a linear inequality to fit a certain situation. However, we must also consider what kind of inequality the question is asking for. The phrase "more than" implies $>$, "less than" implies $<$, "at least" implies $\geq$, and "at most" implies $\leq$.

The main difference between linear equations and linear inequalities is the number of solutions we can obtain. Generally, a linear equation will give us one answer, while a linear inequality will give us a set of values that are the solution.

using techniques of linear programming to model and solve real world problems

Linear programing is a mathematical technique for maximizing or minimizing a linear function of several variables. In linear programming, the decision variables are the variables which will change the output. The objective function is the function that models the goal we are trying to accomplish. For example, if we are considering a company that wants to increase their total profit, the profit will be the objective function. On the decision variables, there are constraints that restrict the variables. There is a non-negativity restriction such that all decision variables should always take non-negative values. For a problem to be a linear programming problem, the decision variables, objective function, and constraints all have to be linear functions.

To solve a linear programming problem, we must first identify the decision variables. Then, we must determine the objective function and determine if we want to find the maximum or minimum. Next, we will have to determine the system of inequalities that the objective function is subject to, or the constraints. Then, we must state the non-negativity restriction. Then, we solve by graphing the equations and inequalities to see where their graphs intersect.

determining connections among proportions, linear functions, and constant rates of change

A linear function is defined as having a constant rate of change throughout the function. This is because the slope of the linear function is consistent for the entire function. The rate of change is the ratio, or proportion, between two related quantities that are changing. The slope of a function is the proportion between the change in outputs to change in inputs. So, the slope is a rate of change of how the outputs of the function are changing compared to how the inputs of the function are changing. Since the slope of a linear function is always constant, linear functions have a constant rate of change throughout the function.

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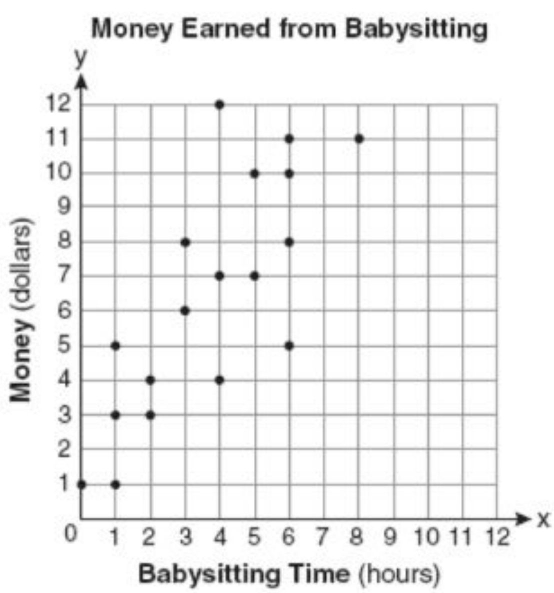
Understanding the Properties of Linear Functions and Relations

1. Find the slope and y-intercept of the linear equation that goes through the points $(-1, -2)$ and $(1,\ 6)$.

2. Write an equation that closely matches the line of best fit for the scatter plot below?

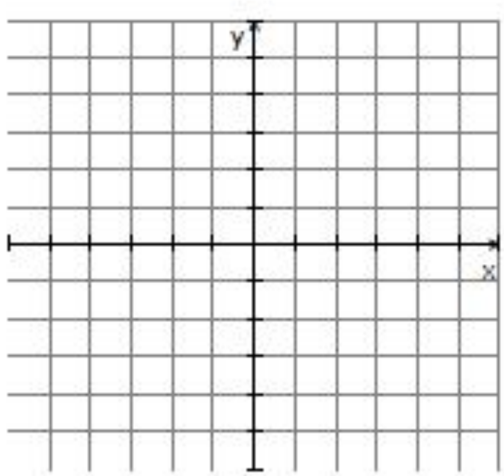


3. Solve the following system of linear equations by elimination, substitution, and a matrix.

$$y + 3x = -4$$
$$2x - 3y = -21$$

4. Graph the following system of linear inequalities:

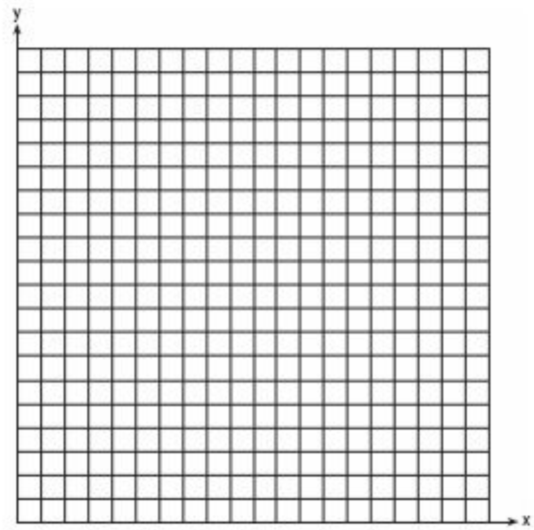
$$2x + 3y \geq 6$$
$$x > y$$



5. A drama club is selling tickets to the spring musical. The auditorium holds 200 people. Tickets cost $12 at the door and $8.50 if purchased in advance. The drama club has a goal of selling at least $1000 worth of tickets to Saturday's show. Write a system of inequalities that can be used to model this scenario. If 50 tickets are sold in advance, what is the minimum number of tickets that must be sold at the door so that the club meets its goal? Justify your answer.

6. The cost of belonging to a gym can be modeled by $C(m) = 50m + 79.50$, where C(m) is the total cost for m months of membership. State the meaning of the slope and y-intercept of this function with respect to the costs associated with the gym membership.

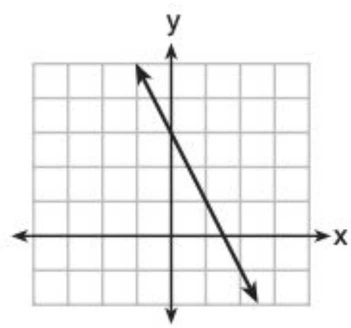
7. A company manufactures bicycles and skateboards. The company's daily production of bicycles cannot exceed 10, and its daily production of skateboards must be less than or equal to 12. The combined number of bicycles and skateboards cannot be more than 16. If x is the number of bicycles and y is the number of skateboards, graph on the accompanying set of axes the region that contains the number of bicycles and skateboards the company can manufacture daily.



8. Which function has a rate of change equal to -3 ? Justify your answer.

x	y
0	2
1	5
2	8
3	11

(1)



(3)

$\{(1,5), (2,2), (3,-5), (4,4)\}$

(2)

$2y = -6x + 10$

(4)

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Understanding the Properties of Linear Functions and Relations

1. Find the slope and y-intercept of the linear equation that goes through the points $(-1,-2)$ and $(1,6)$.

$m = \frac{\Delta y}{\Delta x} = \frac{6 - -2}{1 - -1} = \frac{8}{2} = 4$

$y = 4x + b$

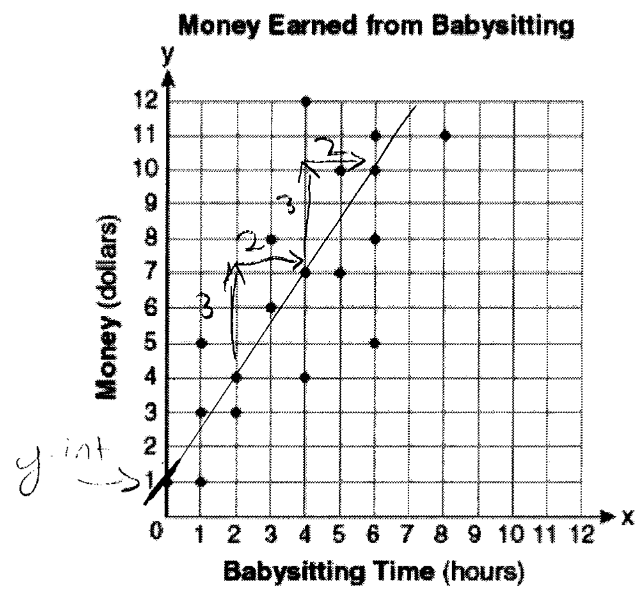
$6 = 4(1) + b$

$6 = 4 + b$

$b = 2$

$y = 4x + 2$

2. Write an equation that closely matches the line of best fit for the scatter plot below?



The line of best fit is about

$y = 3/2x + 2$

3. Solve the following system of linear equations by elimination, substitution, and a matrix.

$$y + 3x = -4$$
$$2x - 3y = -21$$

elimination

3(y + 3x = -4)
2x - 3y = -21

9x + 3y = -12
+ 2x - 3y = -21

11x = -33

x = -3

2(y + 3x = -4)
3(2x - 3y = -21)

2y + 6x = -8
-9y + 6x = -63

11y = 55
y = 5

substitution

y = -3x - 4
2x - 3(-3x - 4) = -21
2x + 9x + 12 = -21
11x = -33
x = -3
y = -3(-3) - 4
y = 5

matrix

$$\left[\begin{array}{cc|c} 1 & 3 & -4 \\ -3 & 2 & -21 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 3 & -4 \\ 0 & -11 & -33 \end{array}\right]$$

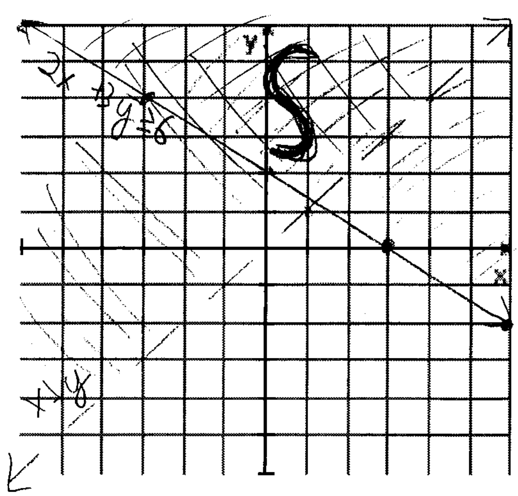
$$\left[\begin{array}{cc|c} 1 & 3 & -4 \\ 0 & 1 & -3 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -3 \end{array}\right]$$

y = 5
x = -3

4. Graph the following system of linear inequalities:

$$2x + 3y \geq 6$$
$$x > y$$



2x + 3y ≥ 6
3y ≥ 6 - 2x
y ≥ 2 - 2/3 x

5. A drama club is selling tickets to the spring musical. The auditorium holds 200 people. Tickets cost $12 at the door and $8.50 if purchased in advance. The drama club has a goal of selling at least $1000 worth of tickets to Saturday's show. Write a system of inequalities that can be used to model this scenario. If 50 tickets are sold in advance, what is the minimum number of tickets that must be sold at the door so that the club meets its goal? Justify your answer.

Let x = the number of tickets sold in advance
Let y = the number of tickets sold at the door

$$\begin{cases} x + y \leq 200 \\ 8.50x + 12y \geq 1000 \end{cases}$$

$8.50(50) + 12y \geq 1000$

$12y \geq 575$

$y \geq 47.917$

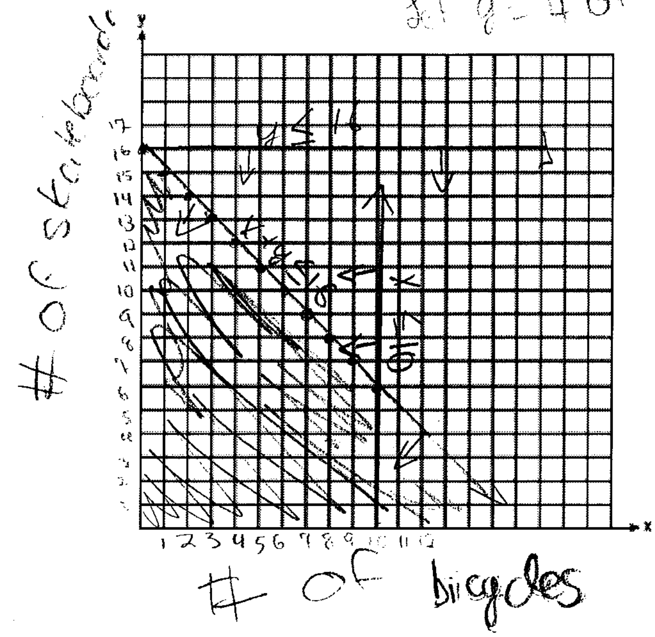
48 tickets must be sold at the door, because 47 tickets would not be enough.

6. The cost of belonging to a gym can be modeled by $C(m) = 50m + 79.50$, where C(m) is the total cost for m months of membership. State the meaning of the slope and y-intercept of this function with respect to the costs associated with the gym membership.

The slope is 50. This represents the amount paid per month for the membership. The y-intercept is 79.50, which represents the initial cost of the membership.

7. A company manufactures bicycles and skateboards. The company's daily production of bicycles cannot exceed 10, and its daily production of skateboards must be less than or equal to 12. The combined number of bicycles and skateboards cannot be more than 16. If x is the number of bicycles and y is the number of skateboards, graph on the accompanying set of axes the region that contains the number of bicycles and skateboards the company can manufacture daily.

Let x = # of bicycles
Let y = # of skateboards



$x + y \leq 16$

$x \leq 10$

$y \leq 12$

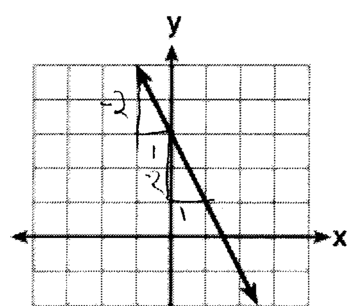
$y \leq -x + 16$

8. Which function has a rate of change equal to -3? Justify your answer.

$\frac{\Delta y}{\Delta x} = 3$

x	y
0	2
1	5
2	8
3	11

(1)



(3)

$\frac{\Delta y}{\Delta x} = -2/1 = -2$

{(1,5), (2,2), (3,−5), (4,4)}

not constant

$2y = -6x + 10$

(4)

$2y = -6x + 10$

$y = -3x + 5$

$m = -3$

2y = −6x+10 has a rate of change of −3 because the slope of the line is −3.