

## Finite Mathematics Problem Sets

For each problem set, please include 5 exercises that you select on the topics of our study. Those will be graded solely on completeness.

### PSA

1.1 13. Let sets  $A, B, C$ , and  $D$  be defined by

$$A = \{x : x \text{ owns a GM car}\}$$

$$B = \{x : x \text{ works for GM}\}$$

$$C = \{x : x \text{ is the president of GM}\}$$

$$D = \{x : x \text{ owns stock in GM}\}$$

Describe in words each of the following sets.

(a)  $A \cap B$       (b)  $B \cap A'$       (c)  $(A \cup B) \cap D$       (d)  $C \cap A$

1.1 22. Counting the empty set and the set itself, how many subsets does each of the following sets contain?

(a)  $\{x\}$       (b)  $\{x, y\}$       (c)  $\{x, y, z\}$       (d)  $\{w, x, y, z\}$

Is there a pattern? If so, what is the pattern? How many subsets does a set with seven elements contain?

1.1 23. Let  $U = \{a, b, c, 2, 4, 6\}$  be a universal set with subsets  $X, Y$ , and  $Z$ . Suppose that  $X \cup Y = \{b, c, 2, 4, 6\}$ ,  $X \cap Y = \{b, 2, 4\}$ ,  $Y' \cap Z' = \{a, c\}$ , and  $Z' = \{a, c, 2\}$ . Find sets  $X, Y$ , and  $Z$  which satisfy these conditions.

1.2 15. Let  $X, A, B$ , and  $C$  be defined by

$$X = \{a, b, c, 1, 2, 3\}$$

$$A = \{a, b, c\} \quad B = \{a, 2, 3\} \quad C = \{1, 2, 3\}$$

Which of the following pairs of subsets form a partition of  $X$ ?

(a)  $A$  and  $B$       (b)  $A$  and  $C$   
(c)  $B$  and  $C$       (d)  $(A \cup B)$  and  $(C \cap B')$

Please provide a Venn diagram of  $X, A, B$ , and  $C$ .

1.2 23. Let  $A, B$ , and  $C$  be distinct subsets of a universal subset  $U$  with  $A \subset B \subset C$ . Also suppose  $n(A) = 3$  and  $n(C) = 7$ . In how many different ways can you select  $B$  such that  $n(B) = 4$ ? Repeat this exercise with  $n(B) = 5$ .

### PSB

1.3 29. Each of the students in the Outdoor Club at Gigantic State University like at least one of the activities of hiking, camping, and canoeing. Of these students, 90 like either hiking or camping or both, 60 like canoeing, and 30 like all three. What can be said about the number of students who like canoeing and exactly one of hiking or camping?

1.3 32. The Transportation and Parking Committee at Gigantic State University collects data from 100 students on how they commute to campus. The following data are obtained:

- 8 drive a car at least part of the time.
- 20 use the bus at least part of the time.
- 48 ride a bicycle at least part of the time.
- 38 do none of these.
- No student who ever drives a car also uses the bus.

How many students who ride a bicycle also drive a car or use the bus?

1.4 24. A student is planning a ski trip. He can go to Colorado or New England. There are 3 possible ski areas for Colorado and 2 times that he can go to each ski area. There are 4 ski areas in New England and 2 times for 2 of the areas but only 1 times for the other 2 areas. A plan involves a location, a ski area, and a time. How many possible plans are there? Please include a tree diagram.

1.4.25. A sociologist can conduct a study in a rural area or in a suburb. If it is conducted in a rural area, it can be done by mail or by telephone, and either a short or along questionnaire can be used. If it is conducted in a suburb, then it can be done by mail, telephone, or personal interview, and either a short or a long questionnaire can be used. How many different plans for a study are there?

2.2 26. There are 3 unfilled roles in a play at the community theater. 2 for females and 1 for a male. Auditioning for the female roles are 4 females including Susan; 3 males audition for the male roles. A cast consists of an assignment of specific people to specific roles.

- (a) How many different casts are there?
- (b) How many different casts include Susan?

For one extra credit point, rewrite and re-solve this problem in a more gender-fluid way (saying more than there are 3 roles and 7 actors).

### PSC

2.1 26. Consider an experiment which consists of planting 10,000 pine tree seeds. Suppose that it is known that about 90 percent of these seeds will germinate and of those that germinate 90 percent will live and grown into seedlings. Define the outcomes of the experiment of planting a single one of these seeds to be:

- (a) The seed does not germinate.
- (b) The seed germinates but dies.
- (c) The seed germinates and grows into a seedling.

What probabilities should be assigned to each of these outcomes?

2.2 29. The time in a television variety show is utilized as indicated as follows, where C denotes commercial and S denotes skit:  $CSSCSCSSC$ . If the producer has 5 skits and 3 commercials, in how many ways can they create a television show? We assume here that a skit is never repeated, but (unfortunately) commercials may appear any number of times.

2.3 24. A student has invitations for job interviews in Atlanta, Boston, Chicago, Detroit, and Elmira.

- (a) In how many ways can they select 3 cities to visit?
- (b) In how many ways can they select 3 cities to visit if both Atlanta and Elmira are included?
- (c) In how many ways can they select 3 cities to visit if not both Atlanta and Elmira are included?

2.3 31. The faculty at GSU is divided into four schools: Humanities, Science, Music and Athletics. The faculty council has 8 members, and it must contain at least one faculty member from each school. In how many different ways (in terms of school representatives) can the council be constituted? (For examples, one possible way is to have 2 members from each of the 4 schools.)

2.4 27. Suppose a basketball team is equally likely to win or lose each game. After 5 games the team has a “record”, i.e. a sequence of wins and losses [note: WLWLW is a “record” here, not 3-2]. What is the probability that there is a string of at least 3 consecutive wins in the team’s record.

### PSD

3.1 15. Fred the weatherman states that on October 30 it will rain with probability .6, there will be a change in the wind direction with probability .7, and there will both rain and a change in the wind direction with probability .5. If Fred is right, what is the probability that either it will rain or there will be change in the wind direction, but not both.

3.1 21-22. Each Monday a student attends mathematics class with probability .6, skips accounting class with probability .3, and attends both with probability .5. Find the probability that they attend at least one class on Monday, and also find the probability that they attend exactly one class.

3.2 20. A student has 7 tickets to a play; 3 are in row A, 2 are in row B, and 2 are in row C. Two tickets are selected at random, and it is noticed that one is *not* in row A.

- (a) Find the probability that both are in row B.
- (b) Find the probability that both are in the same row.

3.2 26. Students at GSU register for courses, and then their schedules are prepared by computer [a fascinating idea to consider]. The computer is programmed to assign each student who registers for a course to a specific section of the course. There are 3 morning sections and 1 afternoon section of accounting and 1 morning section and 1 afternoon section of economics. All these sections are at different times, and the computer assigns students to sections at random. Jose registers for accounting and economics. If he is assigned 1 morning class and 1 afternoon class, what is the probability that he has accounting in the morning?

3.3 22. A group of people who had recently purchased new cars was surveyed about the purchases. Each person was asked to select the single most important factor in the determining their choice and to indicate whether the purchase had proved satisfactory. Of this group, 35 percent cited price as the most important factor, 50 percent cited fuel economy, and the remainder cited styling. Fifty percent of those who cited price as the most important factor expressed satisfaction with their purchase, 80 percent of those who cited fuel economy were satisfied, and 30 percent of those who cited styling were satisfied. View this as an experiment whose outcomes are a reason for purchase (price, fuel economy, styling) and an evaluation of satisfaction (satisfied, not satisfied).

- (a) Draw a tree diagram for this experiment, and find the probability for all outcomes.  
(b) Let  $F$  be the event consisting of all outcomes in which the purchaser was satisfied. Find  $\Pr[F]$ .

#### PSE

3.4 25-26 Pine tree seeds of types W, S, and R are randomly scattered in a field. The seed are 60 percent type W, 30 percent type S, and 10 percent type R. It is known that 30 percent of type W seeds will germinate, 40 percent of type S seeds will germinate, and 70 percent of type R seeds will germinate. If a randomly selected seed has germinated, what is the probability that it is type W? Suppose that a randomly selected seed has not germinated. Find the probability that it is either type W or type S.

3.5 29. A professor who intends to bring their briefcase to the office each morning forgets it one-quarter of the time. Assume that forgetting the briefcase any day always has the same probability, and find the probability that they forget it at least twice a week (out of 5 days).

Newer question: Recent data has estimated the worldwide percentage of Spam emails as 28.5%. A new software company states that their product can detect 98% of actual spam emails as spam. Sometimes (2% of the time), the filter incorrectly labels non-spam emails as spam (false positive). With these percentages in mind, what is the true probability that an email, if labeled spam, is actually a non-spam email?

The last two problems here are at the end of the police activity.

#### PSF

5.1 19. Find the equation of the line through the points (32, 0) and (212, 100), and graph this line. What is the slope of this line? This graph can be related to temperature scales. How should the axes be labeled to illustrate this relationship?

5.1 21. A certain car can be rented at either of two rates:

(a) \$40 per day and \$0.30 per mile driven

(b) \$30 per day and \$0.50 per mile driven

Write equations which describe the costs (in dollars) of driving  $m$  miles in 1 day under each of these rates. Which rate is less expensive for someone who plans to drive 30 miles on a single day?

5.2 15. A rapidly growing suburb has a population of 50,000 and is growing at the rate of 7500 people per year. The adjacent declining city has a population of 1,000,000 and is decreasing at a rate of 125,000 people per year. If these rates continue, in how many years will the population of the suburb equal the population of the city? [Thankfully, these disturbing trends which were very common in the 90s seem to have not continued this way.]

5.2 19. The formula for converting Fahrenheit temperatures to Celsius is  $C = \frac{5}{9}(F - 32)$ . At what temperatures (if any) do the two temperature scale have the same value?

5.2 27. Tom, Dick, and Harry (not any, but these *particular* ones) are being questioned by the police after a robbery at a local fruit market. All three fit the descriptions of the fruit thief, and all three were seen at the market. The police ask each what he bought and how much he paid. Tom says he bought 20 apples and 20 oranges and paid \$8.00. Dick says he bought 15 apples and 5 oranges and paid \$6.75. Harry says he bought 10 apples and 25 oranges and paid \$11.00. After a short pause for computational purposes, the police released Dick and Harry but kept Tom for additional questioning. Why?

### PSG

5.3 4. Raskins and Bobbins Ice Cream Shop makes three kinds of ice cream using skim milk, cream, vanilla, and cacao. Each gallon of Deluxe Vanilla uses 3 quarts of milk, 1 quart of cream, and 2 ounces of vanilla. Each gallon of Regular Vanilla uses 3.5 quarts of milk, 0.5 quarts of cream, and 1 ounce of vanilla. Each gallon of Deluxe Chocolate uses 3.25 quarts of milk, 0.75 quart of cream, and 2 ounces of cacao. How many gallons of each type of ice cream should be made in order to use up 100 gallons of milk, 25 gallons of cream, 5 pounds of vanilla, and 10 pounds of cacao?

5.3. 21. Robin makes bows and arrows using wood, string, and feathers. Each bow uses 5 feet of wood and 4 feet of string, while each arrow uses 3 feet of wood and 4 feathers. If Robin has 100 feet of wood and 32 feet of string, how many feathers does he need so that he can use up all the wood, string, and feathers making bows and arrows?

5.3 28. A student is trying to decide how to allocate her study time among mathematics, English, biology, and economics. She decides to spend a total of 45 hours per week studying and to spend twice as much time on mathematics and biology combined as on English and economics combined. Also, she will spend twice as much time on economics as on English and the same amount of time on mathematics as biology. How much time will she spend on each subject per week?

6.1 15. Let matrices  $A, B,$  and  $C$  be defined by

$$A = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 6 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 2 & 0 \\ 1 & 4 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix}$$

Decide which of the following operations are defined, and carry out those which are defined.

- (a)  $3A - B$  (b)  $CA$   
(c)  $(A - 2B)C$  (d)  $ABC$

6.1.21. Find three  $2 \times 2$  matrices,  $A, B,$  and  $C$  ( $C$  not all zeroes) such that  $AC = BC$  but  $A \neq B$ .

### PSH

6.2 16. (a) Express the following system of equations in matrix form:

$$\begin{aligned} 2x_1 - 2x_2 &= 2 \\ 3x_1 - 2x_3 &= 4 \\ 2x_2 - x_3 &= -2 \end{aligned}$$

- (b) Find the inverse of the coefficient matrix of (a).  
(c) Use the inverse of the coefficient matrix determined in (b) to solve the system of equations.

6.2 17. Let  $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ . Find  $A^{-1}$  and then the inverse of  $A^{-1}$ , denoted by  $(A^{-1})^{-1}$  using the methods of class.

6.2 18. Let  $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$ . Find  $A^{-1}, B^{-1}, (AB)^{-1},$  and  $(BA)^{-1}$ .

6.2 19. Using the results of 18, find  $A^{-1}B^{-1}$  and  $B^{-1}A^{-1}$ . What is the relationship among  $(AB)^{-1}, (BA)^{-1}, A^{-1}B^{-1},$  and  $B^{-1}A^{-1}$ ?

6.2 22. Using matrices  $A$  and  $B$  of 18, and the results of that exercise, decide whether  $(A + B)^{-1}$  is equal to  $A^{-1} + B^{-1}$ . What fact is this related to for real numbers and not matrices?

### PSI

7.1 13. Formulate the following linear programming problem. Be sure to identify the variables, the constraints, and the objective function: The government has mobilized to inoculate the student population against sleeping sickness. There are 200 doctors and 450 nurses available. An inoculation team can consist of either 1 doctor and 3 nurses (called a *full team*) or 1 doctor and 2 nurses (called a *half team*). On average, a full team can inoculate 180 people per hour, while a half team can inoculate 100 people per hour. How many teams of each type should be formed to maximize the number of inoculations per hour?

7.1 19. A toy company makes three monster dolls: Scary Harry, Horrible Harriet, and The Glob. The manufacturing of these dolls is a three-step process: (1) the body is molded from plastic, (2) clothes are put on, and (3) special monster features are added. The amounts of time and material for each step vary from doll to doll, and consequently each doll has its own production cost and associated profit. Data of the manufacturing process are show in the table on the next page. Is the vector which corresponds to the production of 5 Scary Harry, 5 Horrible Harriet, and 10 The Glob dolls feasible? How about the vector which corresponds to 10 Scary Harry, 3 Horrible Harriet, and 2 The Glob dolls?

Doll	Plastic (oz)	Clothes (min)	Features (min)	Profit (\$)
Scary Harry	5	2	3	1.10
Horrible Harriet	3	4	4	1.30
The Glob	10	1	6	2.00
Available	192	55	45	

7.1 27. Formulate the following linear programming problem. Be sure to identify the variables, the constraints, and the objective function: A greenhouse operator plans to bid for the job of providing flowers for the city parks. He will use tulips, daffodils, and flowering shrubs in three types of layouts. A type 1 layout uses 30 tulips, 20 daffodils, and 4 flowering shrubs. A type 2 layout uses 10 tulips, 40 daffodils, and 3 flowering shrubs. A type 3 layout uses 20 tulips, 50 daffodils, and 2 flowering shrubs. The net profit is \$50 for each type 1 layout, \$30 for each type two layout, and \$60 for each type 3 layout. He has 1000 tulips, 800 daffodils, and 100 flowering shrubs. How many layouts of each type should be used to yield the maximum profit?

7.2 23. The manager of a paper-box plant is scheduling the work for one production line for a week. He can produce standard and heavy-duty boxes. Each standard box requires 2 pounds of kraft paper, and each heavy-duty box requires 4 pounds. Also it requires 8 hours of labor to produce 100 standard boxes and 3 hours of labor to produce 100 heavy-duty boxes. (The machine which produces heavy-duty boxes is more efficient.) There are 50 tons of kraft paper and 2400 hours of labor available during the week. Finally, the manager has a contract which requires him to deliver 10,000 heavy-duty boxes at the end of the week. Graphically represent the set of choices available to the manager.

7.2 29-30. Graph the feasible set and find the corner points for the following linear programming problem:

Maximize  $p = 2x + y + 3z$

subject to

$$x \geq 0 \quad y \geq 0 \quad z \geq 0$$

$$3x + 4y + 5z \leq 60$$

$$2x + 3y \leq 0$$

$$x - y \geq 0$$

Solve the linear programming problem. In this special case the problem can be solved directly. Examine the feasible set, and evaluate the objective function on the feasible set.

PSJ

(again there are only four problems here, I think this time I will make them each worth 25% more)

7.3 Solve the linear programming problem formulated in 7.1 13 for the last assignment.

7.3 11. A psychologist plans to conduct an experiment which involves subject who perform activities. After data have been collected, the data are to be analyzed by a team of expert consultants. The psychologist has 15 subject hours available, and they will need to us at least 6 of them. They have funds for a maximum of 200 minutes of consultant time, and each hour of subject time requires at least 30 minutes of consultant time to analyze the data. Depending upon the depth of the analysis, up to 50 minutes of consultant time per subject hour can be profitably used. The information which the psychologist obtains from the experiment depends upon the number of subject hours and the amount of analysis. They estimate that, in appropriate units, 1 unit of information is obtained from each subject hour and 1 unit is obtained from each 25 minutes of consultant analysis. How should the experiment be organized (i.e. how many subject hours and how much consultant analysis) to give the maximum information?

10.1 9. The Kidsports Toy Company makes plastic balls, bats, and rackets for children. The manufacturing of these toys is a three-step process: (1) the plastic is molded, (2) the toy is painted, and (3) special labels are put on the toy. Each ball requires 4 ounces of plastic, 2 minutes of paint time, and 1 minute to apply labels. Each bat requires 9 ounces of plastic, 3 minutes of paint time, and 1 minute to apply labels. Each racket requires 6 ounces of plastic, 2 minutes of paint time, and 3 minutes to apply labels. During each hour that the company's plant is in operation, there are 200 ounces of plastic available, 55 minutes can be used for painting, and 45 minutes can be used for labeling. Also, the company makes \$0.75 per ball, \$1.50 per bat, and \$1.25 per racket. Formulate a linear programming problem whose solution gives the number of balls, bats, and rackets to be manufactured each hour to maximize the company's profit from these toys . Is the problem a SMP?

10.1 33. The supervisor of the Metroburg Street Department makes a choice among a "quick fix", a satisfactory but temporary repair job, and a long-lasting repair job on each city street. A quick fix requires 200 cubic yards of asphalt and 200 hours of labor for each mile of street. A satisfactory but temporary job requires 500 cubic yards of asphalt and 100 hours of labor for each mile, and a long-lasting job requires 1000 cubic yards of asphalt and 200 hours of labor. There are 10,000 cubic yards of asphalt and 1000 hours of labor available. The supervisor estimates that a satisfactory but temporary job has twice the value of a quick fix and a long-lasting job has 4 times the value of a quick fix.

- (a) Formulate a linear programming problem whose solution gives the number of miles of street to be repaired by each method to give maximum value.
- (b) Introduce slack variables and write the inequality constraints as equalities.
- (c) Give an interpretation of each of the slack variables.

PSK

10.2 17. Carry out a pivot operation with the (2) as the pivot element.

	$x$	$y$	$z$	$u$	$v$	Basic Solution
$u$	10	5	8	1	0	80
$v$	3	(2)	12	0	1	30
$p$	-1	-3	2	0	0	0

10.2 26. Find the SMP, i.e. the constraints and the objective function, for the initial tableau in the previous problem.



10.3 7. Identify the entering variable, departing variable, and pivot element of the following tableau:

	$x$	$y$	$z$	$u$	$v$	Basic Solution
$u$	5	0	2	1	$-\frac{1}{3}$	5
$y$	$\frac{1}{2}$	1	0	0	$\frac{1}{6}$	1
$p$	-2	0	-1	0	1	6

10.3 13. Beginning with the previous tableau, use the simplex method to complete the solution of the SMP.

10.3 31. Ralph's Pretty Good Grocery makes two varieties of Raw Bits (the breakfast cereal for above-average people) from oat hulls and pine nuts. A box of Basic Bits uses 6 ounces of oat hulls and 8 ounces of pine nuts, and a box of Lite Bits uses 12 ounces of oat hulls and 4 ounces of pine nuts. Ralph has 2880 ounces of oat hulls and 1248 ounces of pine nuts. He makes a profit of 50 cents on each box of Basic Bits and 30 cents on each box of Lite Bits. How many boxes of each should he make to maximize his profit? [Solve using the simplex method.]