Finite Mathematics Problem Set A Solutions

1.1 13. Let sets A, B, C, and D be defined by

$A = \{x : x \text{ owns a GM car}\}$

$B = \{x : x \text{ works for GM}\}$

$C = \{x : x \text{ is the president of GM}\}$

$D = \{x : x \text{ owns stock in GM}\}$

Describe in words each of the following sets.

(a) $A \cap B$ is the set of all people who own GM cars and work for GM.

(b) $B \cap A'$ is the set of all people who work for GM but do not own GM cars.

(c) $(A \cup B) \cap D$ is the set of all people who own stock in GM and either own GM cars or work for GM.

(d) $C \cap A$ is the set of all people who are the president of GM and own a GM car.

1.1 22. Counting the empty set and the set itself, how many subsets does each of the following sets contain?

(a) $\{x\}$ subsets are only the empty set, $\emptyset$, and the set itself, $\{x\}$, so there are two subsets.

(b) $\{x, y\}$ subsets are $\emptyset, \{x\}, \{y\}, \{x, y\}$, so there are four subsets.

(c) $\{x, y, z\}$ subsets are $\emptyset, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}$, so there are eight subsets.

(d) $\{w, x, y, z\}$ subsets are $\emptyset, \{w\}, \{x\}, \{y\}, \{z\}, \{w, x\}, \{w, y\}, \{w, z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{w, x, y\}, \{w, x, z\}, \{w, y, z\}, \{x, y, z\}, \{w, x, y, z\}$, so there are sixteen subsets.

Is there a pattern? If so, what is the pattern? How many subsets does a set with seven elements contain?

Yes, each time we add an element, the number of subsets doubles. There are 16 subsets for 4 elements. Therefore there are 32 subsets for 5 elements, and 64 subsets for 6 elements. Finally, as requested, there are 128 subsets of 7 elements.

1.1 23. Let $U = \{a, b, c, 2, 4, 6\}$ be a universal set with subsets X, Y, and Z. Suppose that $X \cup Y = \{b, c, 2, 4, 6\}, X \cap Y = \{b, 2, 4\}, Y' \cap Z' = \{a, c\}$, and $Z' = \{a, c, 2\}$. Find sets X, Y, and Z which satisfy these conditions.

For your part, you need to find sets that satisfy these conditions and check that they do, but you might reasonably want to know *how* if you didn't. So, I will try to say something about that. Z is the easiest one, because we are told the elements in the complement, so Z is the other elements, i.e. $\{b, 4, 6\}$.

Finding X and Y takes more work. From the intersection we know that $X = \{b, 2, 4, \dots\}$, $Y = \{b, 2, 4, \dots\}$. The only question then is what is in the dots. The real question is about c and 6, as they are in the union, so the question is are they in X or Y (they can't be in both). Because c is in $Y' \cap Z'$ it is not in Y, since it is in the union it must be in one of them, so c is in X. So far $X = \{b, c, 2, 4, \dots\}, Y = \{b, 2, 4, \dots\}$.

Now what about 6? This is the most interesting part. 6 can either be in X or Y, but not both. Therefore there are two correct answers to this question (you only needed one, and it didn't say there was only one):

First answer: $X = \{b, c, 2, 4, 6\}, Y = \{b, 2, 4\}, Z = \{b, 4, 6\}$.
Second answer: $X = \{b, c, 2, 4\}, Y = \{b, 2, 4, 6\}, Z = \{b, 4, 6\}$.
In both cases, please check that each of the four conditions are met.

1.2 15. Let X, A, B, and C be defined by

$$X = \{a, b, c, 1, 2, 3\}$$
$$A = \{a, b, c\} \qquad B = \{a, 2, 3\} \qquad C = \{1, 2, 3\}$$

Which of the following pairs of subsets form a partition of X?

(a) A and B is not a partition because $A \cap B = \{a\} \neq \emptyset$ and because $A \cup B = \{a, b, c, 2, 3\} \neq X$.

(b) A and C is a partition of X because $A \cap C = \emptyset$ and $A \cup C = X$.

(c) B and C is not a partition because $B \cap C = \{2, 3\} \neq \emptyset$ and because $A \cup B = \{a, 1, 2, 3\} \neq X$.

(d) $(A \cup B)$ and $(C \cap B')$ require computing first (or at least it's easiest that way). $A \cup B = \{a, b, c, 2, 3\} \neq X$ (as done in (a)), and $(C \cap B') = \{1\}$. Notice that these two sets are a partition of X because they are disjoint and their union is X.

Please provide a Venn diagram of X, A, B, and C. Either draw a generic three set Venn diagram in which you identify where each of the 6 elements of X are located, or a "chain" diagram with A and C at opposite ends.

1.2 23. Let A, B, and C be distinct subsets of a universal subset U with $A \subset B \subset C$. Also suppose $n(A) = 3$ and $n(C) = 7$. In how many different ways can you select B such that $n(B) = 4$?

I think the best way to do this is to be explicit. Let's try. Suppose $C = \{p, q, r, s, t, u, v\}$. Suppose $A = \{p, q, r\}$. To build B to have size 4 and contain A we need to add one of $\{s, t, u, v\}$. There are four elements to pick from, so there are four ways.

Repeat this exercise with $n(B) = 5$. Use the same C and A as above. To build B to have size 5 and contain A we need to add two of $\{s, t, u, v\}$. It is not obvious how many ways there are to do this (but we will discuss this soon with "combinations"), so, let's just make a list of the two elements we could add: $\{s, t\}, \{s, u\}, \{s, v\}, \{t, u\}, \{t, v\}, \{u, v\}$, so there are six ways.