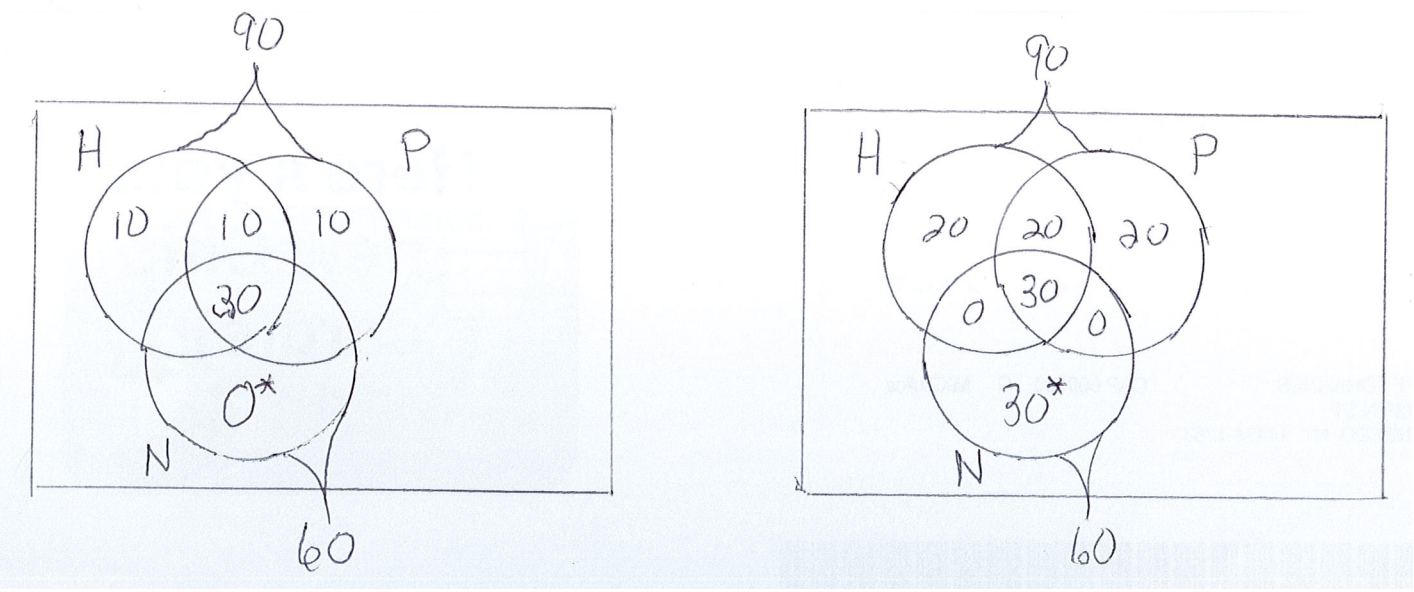
Finite Mathematics Problem Set B Solutions

1.3 29. Each of the students in the Outdoor Club at Gigantic State University like at least one of the activities of hiking, camping, and canoeing. Of these students, 90 like either hiking or camping or both, 60 like canoeing, and 30 like all three. What can be said about the number of students who like canoeing and exactly one of hiking or camping?

The answer is that the number of students who like canoeing and exactly one of hiking or camping is between 0 and 30. I can say that and nothing more. For notation I'm going to let set H be for Hiking, set P for camPing, and set N for caNoeing. We are told $n(N) = 60$. The real area to focus on is the people only in N, i.e. $N \cap H' \cap P'$. Because we know $n(H \cap P \cap N) = 30$, the most that can be in $N \cap H' \cap P'$ is 30 and the least is naturally 0. If $n(N \cap H' \cap P') = 30$, then the number of students who like canoeing and exactly one of hiking or camping is 0. And if $n(N \cap H' \cap P') = 30$, then the number of students who like canoeing and exactly one of hiking or camping is 30. Each of these situations is illustrated in an included diagram. The important number in both cases has an asterisk. Most of the other numbers are not important, only illustrating a possibility.



1.3 32. The Transportation and Parking Committee at Gigantic State University collects data from 100 students on how they commute to campus. The following data are obtained:

8 drive a car at least part of the time.

20 use the bus at least part of the time.

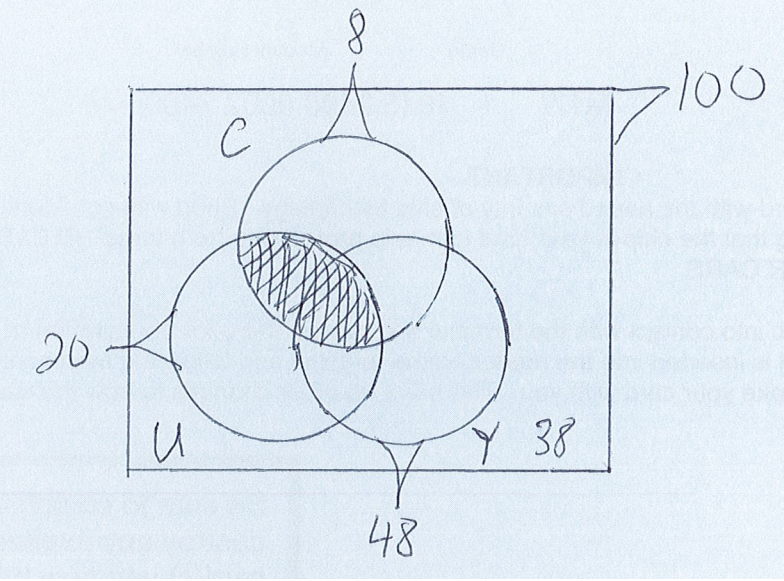
48 ride a bicycle at least part of the time.

38 do none of these.

No student who ever drives a car also uses the bus.

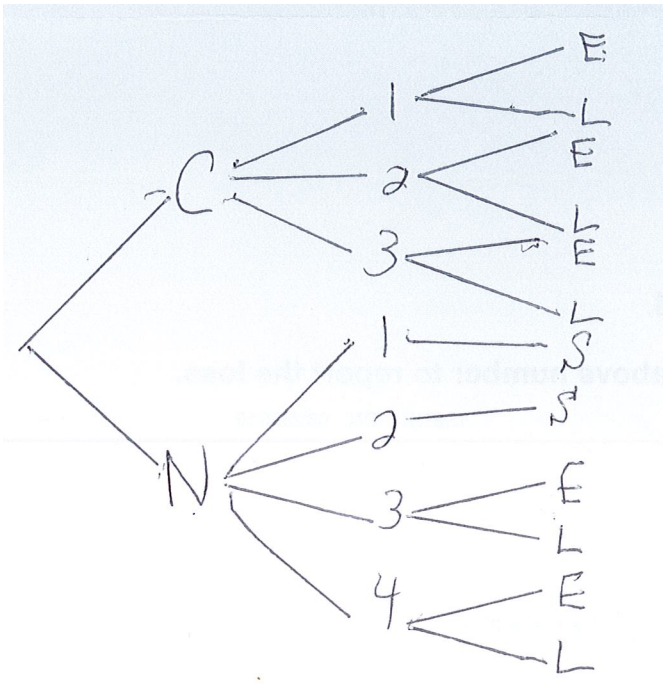
How many students who ride a bicycle also drive a car or use the bus?

I am using C for Car, U for bUs, and Y for bicYcling. Please consider the included diagram. Note that I have market out the region for $C \cap U$. Because there are 100 students total, and 38 on the outside, there are 62 inside the three circles altogether. If there were no overlap among the three sets, there would be $8 + 20 + 48 = 76$ students inside the three circles. The overlap is the students who ride a bicycle and also drive a car or use the bus (because there can be no more overlap since no student who ever drives a car also uses the bus). We don't know how many drive and how many bus, but we know the total must be 14.



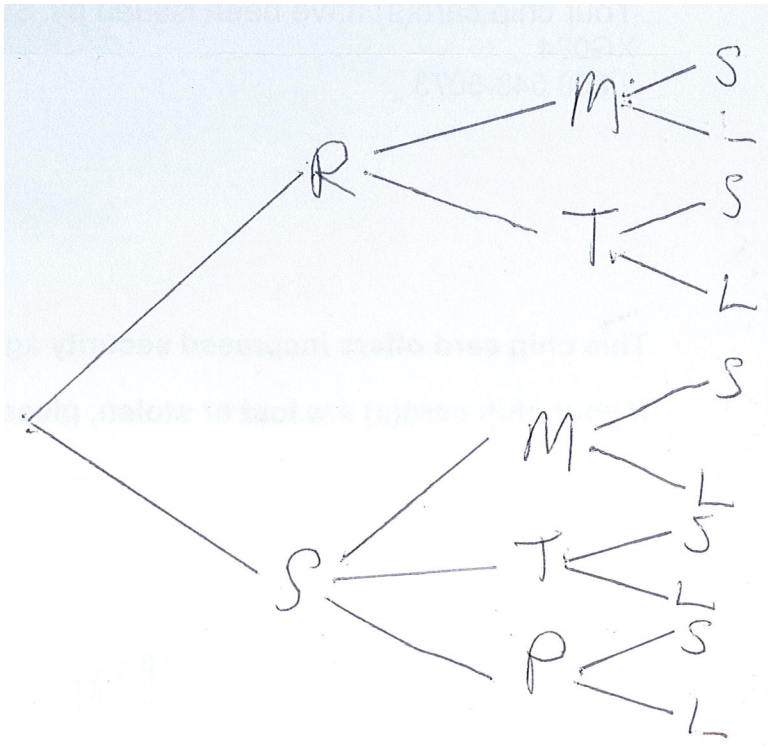
1.4 24. A student is planning a ski trip. He can go to Colorado or New England. There are 3 possible ski areas for Colorado and 2 times that he can go to each ski area. There are 4 ski areas in New England and 2 times for 2 of the areas but only 1 times for the other 2 areas. A plan involves a location, a ski area, and a time. How many possible plans are there? Please include a tree diagram.

Is it me or is it unrealistic that there are more ski areas in New England than in Colorado? Anyway. In my diagram, I am using the following notations: C for Colorado, and N for New England. 1, 2, 3 for the ski areas in Colorado, and 1, 2, 3, and 4 for the ski areas in New England. E for Early time and L for Late time, while S for Standard (or Sole) time. Looking at the "leaves" of the tree, we see there are 12 total plans.



1.4.25. A sociologist can conduct a study in a rural area or in a suburb. If it is conducted in a rural area, it can be done by mail or by telephone, and either a short or a long questionnaire can be used. If it is conducted in a suburb, then it can be done by mail, telephone, or personal interview, and either a short or a long questionnaire can be used. How many different plans for a study are there?

In my diagram, I am using the following notations: R for Rural, and S for Suburb. M for Mail, T for Telephone, and P for Personal Interview. S for Short questionnaire L for Long questionnaire. In the tree diagram you see there are 10 plans.



2.2 26. There are 3 unfilled roles in a play at the community theater. 2 for females and 1 for a male. Auditioning for the female roles are 4 females including Susan; 3 males audition for the male roles. A cast consists of an assignment of specific people to specific roles.
(a) How many different casts are there?
(b) How many different casts include Susan?
For one extra credit point, rewrite and re-solve this problem in a more gender-fluid way (saying more than there are 3 roles and 7 actors).

(a) $4 \cdot 3 \cdot 3 = 36$ (assign the first female, and the second, and then the male).

(b) $2 \cdot 3 \cdot 3 = 1 \cdot 3 \cdot 3 + 3 \cdot 1 \cdot 3 = 18$ (assign Susan to a role, then assign another to the second female role and finally the male; this is the same put Susan in the first role, then someone else in the second role and a male in the male role *or* put someone else in the first role, Susan in the second, and male).

I'll leave the extra part to you.