Finite Mathematics Problem Set C Solutions

2.1 26. Consider an experiment which consists of planting 10,000 pine tree seeds. Suppose that it is known that about 90 percent of these seeds will germinate and of those that germinate 90 percent will live and grown into seedlings. Define the outcomes of the experiment of planting a single one of these seeds to be:
(a) The seed does not germinate.
(b) The seed germinates but dies.
(c) The seed germinates and grows into a seedling.
What probabilities should be assigned to each of these outcomes?

There are 10,000 total seeds. Since 90% germinate, 10% or 1000 do not. Of the remaining 9000, which germinate, 90% or 8100 grow into seedlings, the remaining 10% or 900 die. So, in terms of numbers, 1000 do not germinate, 900 germinate but die, and 8100 germinate and grow into seedlings. Each are out of 10,000 total seeds, so the probabilities are $\frac{1000}{10000} = \frac{1}{10}$, $\frac{900}{10000} = \frac{9}{100}$, and $\frac{8100}{10000} = \frac{81}{100}$.

2.2 29. The time in a television variety show is utilized as indicated as follows, where C denotes commercial and S denotes skit: $CSSCSSC$. If the producer has 5 skits and 3 commercials, in how many ways can they create a television show? We assume here that a skit is never repeated, but (unfortunately) commercials may appear any number of times.

We'll do this in the following order (which is not the run order, but is our planning order). First we will put down the skits, they are most important. There are $5 \cdot 4 \cdot 3 \cdot 2$ ways to do this. Now schedule the commercials, because they can repeat there are $3 \cdot 3 \cdot 3$ ways to do this. Altogether we have $5 \cdot 4 \cdot 3 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 3240$ ways to create the show.

2.3 24. A student has invitations for job interviews in Atlanta, Boston, Chicago, Detroit, and Elmira.
(a) In how many ways can they select 3 cities to visit?
(b) In how many ways can they select 3 cities to visit if both Atlanta and Elmira are included?
(c) In how many ways can they select 3 cities to visit if not both Atlanta and Elmira are included?

(a) $C(5,3) = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10$
(b) 3: we only need to select the third city from BCD.
(c) $10 - 3 = 7$, all ways except those that include both A & E.

2.3 31. The faculty at GSU is divided into four schools: Humanities, Science, Music and Athletics. The faculty council has 8 members, and it must contain at least one faculty member from each school. In how many different ways (in terms of school representatives) can the council by constituted? (For examples, one possible way is to have 2 members from each of the 4 schools.)

Here are the "types": 2,2,2,2; 5,1,1,1; 4,2,1,1; 3,2,2,1; and 3,3,1,1. There is only one way for the first type. There are 4 ways for the second type - pick the one school that has 5 representatives. There are $4 \cdot 3 = 12$ ways for the third type - pick the one school that has 4 representatives then the one remaining school that has two. There are also $4 \cdot 3 = 12$ ways for the fourth type - pick the one school that has 3 representatives then the one remaining school that has one representative (the other schools have 2). Finally, there are $C(4,2) = \frac{4 \cdot 3}{2 \cdot 1} = 6$ ways for the fifth type by picking the two schools (order doesn't matter) that have three representatives. Altogether we have $1 + 4 + 12 + 12 + 6 = 35$.

2.4 27. Suppose a basketball team is equally likely to win or lose each game. After 5 games the team has a "record", i.e. a sequence of wins and losses [note: WLWLW is a "record" here, not 3-2]. What is the probability that there is a string of at least 3 consecutive wins in the team's record.

There are $2^5 = 32$ possible records. Can we count by direct counting the number with at least three consecutive wins? That doesn't seem bad. One is exactly 5 in a row, WWWWW. There are two with exactly 4 in a row: WWWWL and LWWWW. Great, this only leaves those with exactly three in a row, how hard can that be? Ones where the three in a row are first three: WWWLL, WWWLW, only one where it is the middle three LWWWL, and ones where the consecutive are the last three LLWWW WLWWW. That gives $1 + 2 + 5 = 8$. Since each game is equally likely to win or lose, each record is also equally likely, so we have a probability of $\frac{8}{32} = \frac{1}{4}$ that there is a string of at least 3 consecutive wins in the team's record.