**Finite Mathematics Problem Set E Solutions**

3.4 25-26 Pine tree seeds of types W, S, and R are randomly scattered in a field. The seed are 60 percent type W, 30 precent type S, and 10 percent type R. It is known that 30 percent of type W seeds will germinate, 40 percent of type S seeds will germinate, and 70 percent of type R seeds will germinate. If a randomly selected seed has germinated, what is the probability that it is type W? Suppose that a randomly selected seed has not germinated. Find the probability that it is either type W or type S.

Make a tree diagram. By now I hope that is direct and obvious to do. If we denote germinated as G and not germinated as N, then the first question is asking for $Pr[W|G]$. The second question is asking for $Pr[W \cup S|N]$.

Starting on the first question: $Pr[W|G] = \frac{Pr[W \cap G]}{Pr[G]}$. The numerator, $Pr[W \cap G]$ is the probability that the seed is type W and germinates. This is 60% of 30%, i.e. 18%. To find the denominator, $Pr[G]$, we need the probabilities of each of the different ways to germinate, WG, SG, RG. From the tree we find them to be 18% (we just did this one), 12%, and 7%. We add them to find $Pr[G] = 18\% + 12\% + 7\% = 37\%$. So, $Pr[W|G] = \frac{Pr[W \cap G]}{Pr[G]} = \frac{18\%}{37\%} = \frac{18}{37}$.

For the second question: $Pr[W \cup S|N] = \frac{Pr[(W \cup S) \cap N]}{Pr[N]}$. The numerator gives us outcomes WN and SN. From the tree diagram we find their probabilities are 42% and 18% respectively, so $Pr[(W \cup S) \cap N] = 60\%$. For the denominator we also need to include RN, which has probability only 3%. So, $Pr[N] = 63\%$. Thus $Pr[W \cup S|N] = \frac{Pr[(W \cup S) \cap N]}{Pr[N]} = \frac{60\%}{63\%} = \frac{60}{63}$.

3.5 29 A professor who intends to bring their briefcase to the office each morning forgets it one-quarter of the time. Assume that forgetting the briefcase any day always has the same probability, and find the probability that they forget it at least twice a week (out of 5 days).

At least twice a week is 2, 3, 4, or 5 times. It is also not 0 or 1 time. We can do this both ways. 2, 3 4 or 5 is: $C(5,2)(\frac{1}{4})^2(\frac{3}{4})^3 + C(5,3)(\frac{1}{4})^3(\frac{3}{4})^2 + C(5,4)(\frac{1}{4})^4(\frac{3}{4})^1 + C(5,5)(\frac{1}{4})^5(\frac{3}{4})^0$. Not 0 or 1 time is: $1 - C(5,0)(\frac{1}{4})^0(\frac{3}{4})^5 - C(5,1)(\frac{1}{4})^1(\frac{3}{4})^4$. Both ways we get $\frac{47}{128} = 0.3671875$.

"Newer question" Recent data has estimated the worldwide percentage of Spam emails as 28.5%. A new software company states that their product can detect 98% of actual spam emails as spam. Sometimes (2%) of the time), the filter incorrectly labels non-spam emails as spam (false positive). With these percentages in mind, what is the true probability that an email, if labeled spam, is actually a non-spam email? To start with the worldwide percentage of non-spam is 71.5%. We're looking for Pr(non-spam | labeled spam). So we want Pr(non-spam $\cap$ labeled spam) = 0.715*0.02 = 0.0143. Pr(labeled spam) = Pr(non-spam $\cap$ labeled spam) + Pr(spam $\cap$ labeled spam)= $0.0143 + 0.285 * 0.98 = 0.2936$. So Pr(non-spam | labeled spam) = Pr(non-spam $\cap$ labeled spam)/Pr(labeled spam) = 0.0143/0.2936 = 0.048706 = 4.9%, so that's just under 5% of your email that you don't see that isn't spam.

First policing question: Pr(searched | motorist Black) = searched and black / black stopped = $\frac{67985}{1165871} \simeq 0.058313 \simeq 5.8\%$. Pr(searched | motorist White) = searched and white / white stopped = $\frac{47826}{1670873} \simeq 0.028623 \simeq 2.9\%$. Almost exactly twice as much. That seems enough to be concerning.

Second policing question - this is your chance to be thoughtful without doing any computation. Sincere thoughtful answers will earn full credit.