Finite Mathematics Problem Set F Solutions

5.1 19. Find the equation of the line through the points (32, 0) and (212, 100), and graph this line. What is the slope of this line? This graph can be related to temperature scales. How should the axes be labeled to illustrate this relationship?

Let's do slope first. the slope is $\frac{100-0}{212-32} = \frac{100}{180} = \frac{5}{9}$. Regarding temperature, I am hopeful that you recognise that 32F is freezing and 0C is freezing. Perhaps you also recognise that the boiling point for water is 212F and 100C. If you put all this together, you see this is conversion between Fahrenheit and Celcius temperature scales. Therefore the first coordinate should be labeled F and the second should be C. For graphing, please ensure that you have a line through the two given points. That should suffice. I think this only leaves the equation. There are several ways to get there, I'll try this one: $C = \frac{5}{9}F + b$ for some b. We also know it needs to work for $(32, 0)$. So, $0 = \frac{5}{9}(32) + b$, from this $0 = \frac{160}{9} + b$, so $b = -\frac{160}{9}$. I will continue trying to be extra thorough on solutions since it's one of the few ways I have to convey mathematics to you. This produces $C = \frac{5}{9}F - \frac{160}{9}$. It is worth checking that it works for the other point also $\frac{5}{9}(212) - \frac{160}{9} = \frac{1060-160}{9} = \frac{900}{9} = 100$ as we hoped. This could also be written as $\frac{5}{9}F - C = \frac{160}{9}$ or $5F - 9C = 160$.

5.1 21. A certain car can be rented at either of two rates:
(a) \$40 per day and \$0.30 per mile driven
(b) \$30 per day and \$0.50 per mile driven
Write equations which describe the costs (in dollars) of driving m miles in 1 day under each of these rates. Which rate is less expensive for someone who plans to drive 30 miles on a single day?

For the first option we have $r_a = 40 + 0.3m$ and for the second option we have $r_b = 30 + 0.5m$. Now we check what the rates are for 30 miles $r_a = 40 + 0.3(30) = \$49$ and $r_b = 30 + 0.5(30) = \$45$. So, the second rate is less expensive.

5.2 15. A rapidly growing suburb has a population of 50,000 and is growing at the rate of 7500 people per year. The adjacent declining city has a population of 1,000,000 and is decreasing at a rate of 125,000 people per year. If these rates continue, in how many years will the population of the suburb equal the population of the city? [Thankfully, these disturbing trends which were very common in the 90s seem to have not continued this way.]

For the suburb, we have $p = 50000 + 7500t$ where p is the population and t is the time in years. For the city we have $p = 1000000 - 125000t$. The question asks when will the two populations equal, so we set them equal: $50000 + 7500t = 1000000 - 125000t$. Adding $125000t$ to each side produces $50000 + 132500t = 1000000$. Then subtracting 50000 from each side produces $132500t = 950000$. Finally dividing both by 1325000 produces $t = \frac{950000}{132500}$. This is over 7 years, approximately 7.17 years.

5.2 19. The formula for converting Fahrenheit temperatures to Celsius is $C = \frac{5}{9}(F - 32)$. At what temperatures (if any) do the two temperature scale have the same value?

This should be familiar from the first problem, although this isn't a form that we found it in, but please notice that if you distribute the right side you do get $\frac{5}{9}F - \frac{160}{9}$ as before. The question asks when $C = F$, or when $F = \frac{5}{9}(F - 32)$. To avoid fractions we can multiply both sides by 9 and distribute the right side to get $9F = 5F - 160$ Then we can subtract $5F$ from both sides to get $4F = -160$. Finally we divide each by for 4 to produce $F = C = -40$. So, when it is -40 either Fahrenheit or Celsius is the same. I think they call that temperature "cold" in both.

5.2 27. Tom, Dick, and Harry (not any, but these *particular* ones) are being questioned by the police after a robbery at a local fruit market. All three fit the descriptions of the fruit thief, and all three were seen at the market. The police ask each what he bought and how much he paid. Tom says he bought 20 apples and 20 oranges and paid \$8.00. Dick says he bought 15 apples and 5 oranges and paid \$6.75. Harry says he bought 10 apples and 25 oranges and paid \$11.00. After a short pause for computational purposes, the police released Dick and Harry but kept Tom for additional questioning. Why?

One more question. This one is the most interesting, and I think you found that. "Why?" is the hard part. The question is really how the different stories fit together. If the stories don't fit together, then that's suspicious. Let's look at each of them.

Tom: $20a + 20o = 8$

Dick: $15a + 5o = 6.75$

Harry: $10a + 25o = 11$.

I am going to do all of this using elimination and reduction to solve these equations, mostly because we don't have practice with it in any of the other problems. Please be aware of this method. First we consider how Tom and Dick's stories fit together. $20a+20o = 8$ and $15a + 5o = 6.75$. If we multiply the second equation by 4 it produces $60a + 20o = 27$, and still a true equation. We may subtract $20a + 20o = 8$ from $60a + 20o = 27$ to get $40a = 19$ and this produces $a = 0.475$, which is a little strange having apples cost 47.5¢ each. But, what about oranges? If $a = 0.475$, then using Tom's story we have $20(0.475) + 20o = 8$ so $9.5 + 20o = 8$. But, then things get strange if we subtract 9.5 from each side to get $20o = -1.5$ and $o = -0.075$, and that's really strange. I do not think the market pays people 1.5¢ to take oranges. So, these stories do not fit together.

Let's try Dick and Harry: $15a + 5o = 6.75$ and $10a + 25o = 11$. So that we can cancel the oranges, we will multiply the first equation by 5 to get $75a + 25o = 33.75$. Now subtract $10a + 25o = 11$ from $75a + 25o = 33.75$ to get $65a = 22.75$, and divide both sides by 65 to get $a = 0.35$, or apples at 35¢ each. That seems reasonable. What would this say about oranges? Using Harry's story we have $10(0.35) + 25o = 11$ so $3.5 + 25o = 11$. Subtract 3.5 from each side to get $25o = 7.5$. Then divide by sides by 25 and $o = 0.30$, so oranges are 30¢ each. That sounds completely reasonable. Dick and Harry's stories seem to fit nicely together. This means that perhaps Tom is the suspicious one.

Let's see how Tom fits with Harry. If he doesn't then I think we've settled the case. So, now let's compare Tom and Harry: $20a+20o = 8$ and $10a+25o = 11$. To cancel the apples, we multiply the second equation by 2 to get $20a + 50o = 22$. Now subtract $20a + 20o = 8$ from $20a + 50o = 22$ to get $30o = 14$. Dividing both sides by 30 gives $o = 0.466666\ldots$, which says oranges are $46\frac{2}{3}$¢ each. That seems strange again. What do we get for apples? This is harder with the strange numbers, but returning to Tom's suspicious story we get $20a + 20(0.4666\ldots) = 8$. From this we compute to find $20a + 9.333\cdots = 8$, and then things get really strange when we subtract $9.333\ldots$ from both sides giving $20a = -1.333\ldots$ and thus$a = -0.0666\ldots$ or the market pays people $6\frac{2}{3}$¢ to take apples. And that doesn't make any sense.

Dick and Harry's stories give us entirely reasonable numbers. They fit together well. Tom's story doesn't fit with either of the others. We have our thief!

To be clear - to earn full credit on this problem, you need to analyse each pair together, as I have done.