Finite Mathematics Problem Set I Solutions

PSI

7.1 13. Formulate the following linear programming problem. Be sure to identify the variables, the constraints, and the objective function: The government has mobilized to inoculate the student population against sleeping sickness. There are 200 doctors and 450 nurses available. An inoculation team can consist of either 1 doctor and 3 nurses (called a *full team*) or 1 doctor and 2 nurses (called a *half team*). On average, a full team can inoculate 180 people per hour, while a half team can inoculate 100 people per hour. How many teams of each type should be formed to maximize the number of inoculations per hour?

Let's say that a full team is called f, and a half team is called h. We know $f \geq 0$ and $h \geq 0$ because we can't have negative teams. We also have an inequality about doctors which is $f + h \leq 200$, and an inequality about nurses $3f + 2h \leq 450$. The goal is to maximize inoculations, i.e. $i = 180f + 100h$.

To be explicit, the variables are f and h because we want to know the number of teams of each kind. The constraints are:

$$f \geq 0, h \geq 0, f + h \leq 200, 3f + 2h \leq 450$$

And the objective function is $i = 180f + 100h$.

7.1 19. A toy company makes three monster dolls: Scary Harry, Horrible Harriet, and The Glob. The manufacturing of these dolls is a three-step process: (1) the body is molded from plastic, (2) clothes are put on, and (3) special monster features are added. The amounts of time and material for each step vary from doll to doll, and consequently each doll has its own production cost and associated profit. Data of the manufacturing process are show in the table on the next page. Is the vector which corresponds to the production of 5 Scary Harry, 5 Horrible Harriet, and 10 The Glob dolls feasible? How about the vector which corresponds to 10 Scary Harry, 3 Horrible Harriet, and 2 The Glob dolls?

Doll	Plastic (oz)	Clothes (min)	Features (min)	Profit ($)
Scary Harry	5	2	3	1.10
Horrible Harriet	3	4	4	1.30
The Glob	10	1	6	2.00
Available	192	55	45	

The variables are how many dolls are produced, let's say s for Scary Harry, h for Horrible Harriet, and g for The Glob. We have natural constraints that none of them may be negative, but that doesn't really affect the answer here. We also have constraints of plastic, time for clothes, and time for features. For plastic we have the constraint $5s + 3h + 10g \leq 192$, for clothes we have $2s + 4h + g \leq 55$, and for features the constraint is $3s + 4h + 6g \leq 45$.

To check if the particulars are possible, we verify if they satisfy the constraints: $5(5) + 3(5) + 10(10) = 140 \leq 192$. This works. Next $2(5) + 4(5) + 10 = 40 \leq 55$. This also works.

$3(5)+4(5)+6(10)=95>45$. This isn't even close to working. The time spent on the globs themselves is over the allotted 45 minutes. What were they thinking?

Ok, we check the other ones. $5(10)+3(3)+10(2)=79\leq 192$. This works. Next $2(10)+4(3)+2=34\leq 55$. This also works. Did we fix the feature problem? $3(10)+4(3)+6(2)=54>45$. Curses, no, this still does not work. I give up. Let's stop making monster dolls.

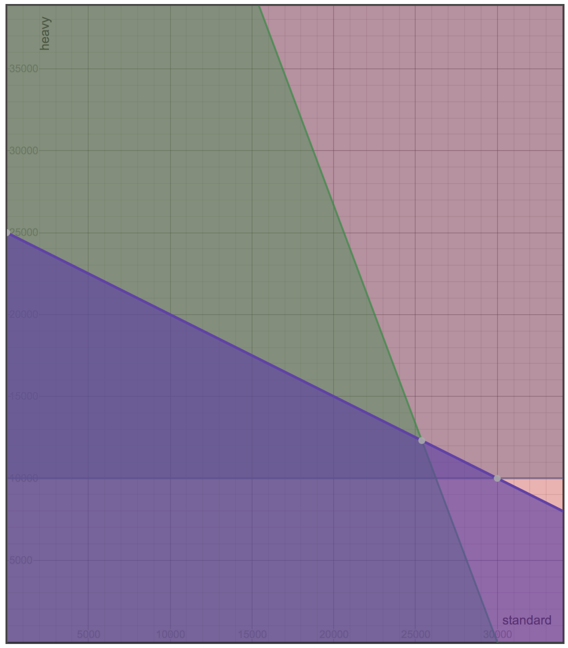
7.1 27. Formulate the following linear programming problem. Be sure to identify the variables, the constraints, and the objective function: A greenhouse operator plans to bid for the job of providing flowers for the city parks. He will use tulips, daffodils, and flowering shrubs in three types of layouts. A type 1 layout uses 30 tulips, 20 daffodils, and 4 flowering shrubs. A type 2 layout uses 10 tulips, 40 daffodils, and 3 flowering shrubs. A type 3 layout uses 20 tulips, 50 daffodils, and 2 flowering shrubs. The net profit is \$50 for each type 1 layout, \$30 for each type two layout, and \$60 for each type 3 layout. He has 1000 tulips, 800 daffodils, and 100 flowering shrubs. How many layouts of each type should be used to yield the maximum profit?

The variables are for the numbers of each layout type. It's a bit awkward, but I'll use the second letter (because you don't want me using 1 as a variable), so we have n of type oNe, w of type tWo, and h of type tHree. As always, we have $n\geq 0$, $w\geq 0$, and $h\geq 0$. Furthermore there are constraints for tulips, daffodils, and shrubs. The tulip constraint is $30n+10w+20h\leq 1000$. The daffodil constraint is $20n+40w+50h\leq 800$. And the shrub constraint is $4n+3w+2h\leq 100$. The objective is to maximise profit, which is given by $p=50n+30w+60h$. Actually solving this problem with three variables requires some significant visualisations. Fortunately we're not asked for this.

7.2 23. The manager of a paper-box plant is scheduling the work for one production line for a week. He can produce standard and heavy-duty boxes. Each standard box requires 2 pounds of kraft paper, and each heavy-duty box requires 4 pounds. Also it requires 8 hours of labor to produce 100 standard boxes and 3 hours of labor to produce 100 heavy-duty boxes. (The machine which produces heavy-duty boxes is more efficient.) There are 50 tons of kraft paper and 2400 hours of labor available during the week. Finally, the manager has a contract which requires him to deliver 10,000 heavy-duty boxes at the end of the week. Graphically represent the set of choices available to the manager.

Our variables here are s for standard and h for heavy duty boxes. Naturally $s,h\geq 0$. We have a constraint for paper: $2s+4h\leq 50(2000)$ (remember there are 2000 pounds in a ton). We also have a constraint for labour: $\frac{8}{100}s+\frac{3}{100}h\leq 2400$. And there is one more constraint, not only do we need $h\geq 0$, but in fact we need $h\geq 10000$.

You graph should look something similar to this.



I put standard on the horizontal axis. You may have switched them. If we were solving this problem, we would find the four corners and check the objective function there. We're clearly not ... because there is no objective function given.

7.2 29-30. Graph the feasible set and find the corner points for the following linear programming problem:
Maximize $p = 2x + y + 3z$
subject to

$$x \geq 0 \qquad y \geq 0 \qquad z \geq 0$$

$$3x + 4y + 5z \leq 60$$

$$2x + 3y \leq 0$$

$$x - y \geq 0$$

Solve the linear programming problem. In this special case the problem can be solved directly. Examine the feasible set, and evaluate the objective function on the feasible set.

You must know there is something sneaky here. We wouldn't usually be asked to graph in three dimensions and to go through all the work. Here's the catches: We know $x, y \geq 0$. But also, $2x + 3y \leq 0$. The only way that can be less than zero is if one is negative, which is prohibited, and the only way it can equal zero is therefore if both x and y are zero. Ok, so, now we have $x = y = 0$ and $z \geq 0$ and $5z \leq 60$. The second is the same as $z \leq 12$. Therefore we have $x = y = 0$ and $0 \leq z \leq 12$ for our feasible set. If you graph it, it is a line segment from 0 to 12. Our objective function, $p = 2x + y + 3z$ is smallest when $z = 0$, where $p = 0$, and largest where $z = 12$, giving a maximum value of $p = 36$. So, we have found the maximum possible value.