EXAM I – VERSION 1

1. Let A, B, and C be subsets of a universal set U. Suppose $n(\mathrm{A}) = \mathrm{n}(\mathrm{B}) = 25$, $n(\mathrm{A} \cap \mathrm{B}) = 15$, $n(\mathrm{A} \cap \mathrm{C}) = 10$, $n(\mathrm{B} \cap \mathrm{C}) = 12$, $n(\mathrm{A}' \cap \mathrm{B}' \cap \mathrm{C}') = 20$, $n(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}) = 3$, and $n(\mathrm{U}) = 80$. Find $n(\mathrm{C})$.

   (a) 43 (b) 8 (c) 2 (d) 25 (e) none of the others

2. In a multi-step experiment, the first step is to roll a die and note the outcome (1,2,3,4,5, or 6). Next, if the result on the die is even, then the die is rolled again, the result is noted, and the experiment ends. If the result on the first die is odd, then a coin is flipped twice, the *number of heads* is noted, and the experiment ends. Let S be the sample space for this experiment. Find $n(\mathrm{S})$.

   (a) 36 (b) 24 (c) 27 (d) 30 (e) none of the others

3. Suppose each student in M118 is to be assigned a code number which starts with a letter from the set $\{A, B, C, D, E, F\}$ and then is followed by 3 digits each from the set $\{1, 2, 3, 4, 5, 6\}$. For example, B-3-4-3 is one such code. How many such codes exist?

   (a) $6 \times P(7,3)$ (b) 6^4 (c) 6×7^3 (d) $6 \times C(7,3)$ (e) none of the others

4. An unfair coin is weighted so that "heads" come up *one third* as often as "tails". What probability should be assigned to the outcome "heads" to reflect this weighting?

   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{5}$ (e) none of the others

5. An experiment consists of first picking two names from a list of six names, then picking two jobs from a list of 5 jobs, and finally picking two cities from a list of 4 cities. How many outcomes are there for this experiment?

   (a) $C(6,2) + C(5,2) + C(4,2)$ (b) 7200
   (c) $P(6,2) + P(5,2) + P(4,2)$ (d) 900
   (e) none of the others.

6. How many 5 letter "words" can be formed from a set of 8 letters if no letter can be used more than once?

   (a) $C(8,5)$ (b) 1680 (c) 5^8 (d) $P(8,5)$ (e) none of the others

7. Two fair dice are rolled and the *product* of the two numbers is noted. What is the probability that the product is *less than* 25?

(a) $\frac{8}{9}$ (b) $\frac{4}{12}$ (c) $\frac{1}{9}$ (d) $\frac{7}{9}$ (e) none of the others

8. How many subsets with 7 elements are in a set with 9 elements?

(a) $P(7,5)$ (b) $P(7,2)$ (c) 72 (d) 36 (e) none of the others

9. A purse contains 3 nickels and 5 dimes. Three coins are selected at random from the purse. What is the probability that the value of the coins is 25 cents?

(a) $\frac{15}{28}$ (b) $\frac{5}{8}$ (c) $\frac{13}{28}$ (d) $\frac{3}{8}$ (e) none of the others

10. A committee contains 5 men and 6 women. Two members are needed to fill the offices of chair and vice-chair. How many ways can these offices be filled if both office holders cannot be the same sex?

(a) 60 (b) $P(6,2)+P(5,2)$ (c) 120
(d) $C(11,2)-C(6,2)-C(5,2)$ (e) none of the others

11. A psychology major is required to take 2 core biology courses and 2 core mathematics courses. If there are 4 approved biology courses and 6 approved mathematics courses, how many choices are available to satisfy these requirements?

(a) $P(4,2)\cdot P(6,2)$ (b) 45 (c) $C(4,2)+C(6,2)$
(d) $C(6,2)\cdot C(4,2)$ (e) none of the others

12. A true-false test has 10 questions. Sam *knows* the answers to 8 of the questions, and he *guesses* (probability of being correct $=\frac{1}{2}$) on each of the other two questions. What is the probability that he gets all 10 answers correct?

(a) $\frac{1}{2^{10}}$ (b) $\frac{C(10,2)}{P(10,2)}$ (c) $\frac{1}{2^2}$ (d) $\frac{1}{8\cdot 2^2}$ (e) none of the others

13. An experiment, with equally likely outcomes, has a sample space S and an event E with $n(\mathrm{S})=50$ and $Pr[\mathrm{E}]=.3$. What is $n(\mathrm{E})$?

(a) 20 (b) 45 (c) 15 (d) 30 (e) none of the others

14. Two fair dice are rolled and the numbers on the two dice are noted. What is the probability that these numbers are different?

(a) $\frac{25}{36}$ (b) $\frac{1}{2}$ (c) $\frac{1}{36}$ (d) $\frac{5}{6}$ (e) none of the others

15. An experiment has the sample space $S = \{\sigma_1, \sigma_2, \sigma_3\}$. It is known that σ_1 and σ_2 occur equally often and σ_3 occurs three times as often as σ_1. What is $w_2 = Pr[\{\sigma_2\}]$?

    (a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$ (e) none of the others

16. Let $U = \{1, 2, 3, \ldots, 15\}$ be a universal set with subsets $E = \{2, 4, 6, 8\}$, $F = \{1, 3, 5, 7, 9\}$, and $G = \{7, 8, 9, 10, 11, 12\}$. What is $n((E \cup F)' \cap G')$?

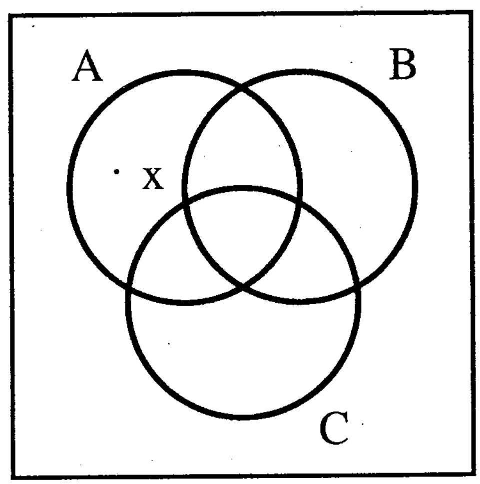
    (a) 0 (b) 3 (c) 2 (d) 1 (e) none of the others

17. Let A and B be subsets of a universal set U. Which of the following is *always true*?

    (a) $A \cap B' = A' \cap B$
    (b) $A \cap B = (B \cup A)'$
    (c) $(A \cup B)' = A' \cap B'$
    (d) $(A \cup B)' = A' \cup B'$
    (e) none of the others

18. Which of the following is a *true* statement about the point x in the Venn diagram to the right?

    (a) $x \in A \cap B$
    (b) $x \in B \cup C$
    (c) $x \in A' \cap B$
    (d) $x \notin B \cup C$
    (e) none of the others



19. Let A and B be subsets of a universal set U. Suppose $n(A \cap B) = 5$, $n(A' \cap B') = 10$, $n(A' \cap B) = 10$, and $n(A \cup B) = 28$. Find $n(U)$.

    (a) 18 (b) 38 (c) 48 (d) 18 (e) none of the others

20. The 100 residents of Hickory, Indiana are asked if they like to watch basketball and/or football. The results are that 87 like to watch basketball, 21 like to watch football, and 8 *dislike* both. How many like to watch both?

    (a) 97 (b) 8 (c) 15 (d) 11 (e) none of the others

EXAM I – VERSION 2

1. (5 points) A and B are subsets of a universal set U such that $n(A) = 58$, $n(B) = 52$, $n((A \cup B)') = 100$, and $n(U) = 177$. How many elements of U are in exactly one of the subsets A and B?

(A)	110	(B)	33	(C)	58	(D)	52	(E)	44
(F)	25	(G)	77	(H)	19	(J)	34	(K)	none of the others

2. (5 points) A cookie tray has 8 chocolate chip cookies, 11 oatmeal cookies, and 9 sugar cookies. If one cookie is drawn at random, what is the probability that it is a sugar cookie?

(A)	11/792	(B)	11/28	(C)	9/28	(D)	1/3	(E)	1/792
(F)	1/28	(G)	9/792	(H)	8/28	(J)	8/792	(K)	9/3,276
(L)	none of the others								

3. (5 points) A small furniture store has 4 sofas, 6 floor lamps, and 3 coffee tables on display. If Anne buys one of each to furnish her living room, how many different outcomes of a sofa, a floor lamp, and a coffee table are possible?

(A)	286	(B)	3	(C)	1,716	(D)	373,248	(E)	1
(F)	72	(G)	27	(H)	59,640	(J)	357,840	(K)	13
(L)	2,197	(M)	none of the others						

4. (5 points) Dan and Mary are taking their three children to the Nutcracker Ballet. They have tickets for seats H101–105. In how many different ways can the family sit in these seats if each person sits in one of these seats and no one sits in someone's lap?

(A)	3,125	(B)	25	(C)	15	(D)	48	(E)	120
(F)	240	(G)	10	(H)	60	(J)	1	(K)	none of the others

5. (5 points) A shelf of books at the bookstore has 4 different detective novels, 5 different science fiction books, and 4 different spy novels. If John selects 3 of these at random, how many different outcomes are possible?

(A)	286	(B)	1,716	(C)	3	(D)	72	(E)	357,840
(F)	80	(G)	1	(H)	59,640	(J)	13	(K)	373,248
(L)	2,197	(M)	27	(N)	none of the others				

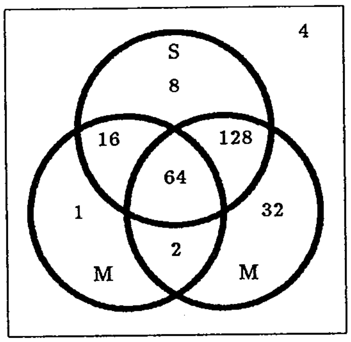
6. (5 points) Joe, Anne, and Mike each select one of the 9 plays produced by the Theatre Department to attend. If someone records who is going to which play, how many different outcomes are possible?

(A)	27	(B)	2,187	(C)	504	(D)	84	(E)	512
(F)	24	(G)	729	(H)	9	(J)	19,683		
(K)	none of the others								

7. (5 points) A survey of a group of 870 IU freshmen showed that 135 had attended a football game here, 110 had attended a women's basketball game here, 90 had attended a soccer game here, 35 had attended a football game and a women's basketball game here, 25 had attended a football game and a soccer game here, 15 had attended a women's basketball game and a soccer game here, and 10 had attended all three types of games here. How many had attended none of these types of games here?

(A)	450	(B)	600	(C)	510	(D)	360	(E)	500
(F)	480	(G)	420	(H)	520	(J)	270		
(K)	none of the others								

8. (5 points) A group of investors has been classified according to whether they own stocks (S), bonds (B), or mutual funds (M). The results are presented in the diagram below. How many own exactly one of these three types of investments or do not own both stocks and bonds.



(A)	249	(B)	191	(C)	253
(D)	175	(E)	239	(F)	235
(G)	187	(H)	171	(J)	127
(K)	123	(L)	45	(M)	150
(N)	none of the others				

9. (5 points) Dorm A has 5 single rooms (i.e., only one person per room) available and Dorm B has 6 single rooms available. If Sue, Ann, and Julie are each assigned one of these available rooms, how many different outcomes are possible?

(A)	165	(B)	4,060	(C)	1,331	(D)	27	(E)	11
(F)	990	(G)	30	(H)	24,360	(J)	1	(K)	33
(L)	3	(M)	90	(N)	none of the others				

10. (5 points) A, B, and C are subsets of a universal set U such that $n(U) = 230$, $n(A) = 58$, $n(B) = 35$, $n(C) = 34$, $B \cap C = \emptyset$, and $n((A \cup B \cup C)') = 130$. How many members of U are in exactly one of the sets A, B, and C?

(A)	27	(B)	127	(C)	66	(D)	130	(E)	73
(F)	31	(G)	42	(H)	65	(J)	230	(K)	insufficient information
(L)	none of the others								

11. (5 points) A cage contains 5 white mice, 7 grey mice, and 4 brown mice. If 3 brown mice and 2 grey mice are chosen at random for an experiment, how many different outcomes are possible?

(A)	84	(B)	4,368	(C)	1,008	(D)	70	(E)	210
(F)	2,520	(G)	25	(H)	232	(J)	524,160	(K)	3,136
(L)	113	(P)	none of the others						

12. (5 points) Which of the following assignments of truth values make the sentence $p \vee \sim (q \to r)$ true?

(I) $p = F,\ q = T,\ r = T$ (III) $p = T,\ q = T,\ r = F$
(II) $p = F,\ q = T,\ r = F$ (IV) $p = F,\ q = F,\ r = F$

(A) None (B) IV only (C) II and III only (D) I, II, and IV only
(E) I only (F) I and II only (G) II and IV only (H) I, III, and IV only
(J) II only (K) I and III only (L) III and IV only (M) II, III, and IV only
(N) III only (P) I and IV only (Q) I, II, and III only (R) I, II, III, and IV
(S) none of the others

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13. (5 points) A sandwich buffet has 6 different types of meat, 4 different types of cheese, and 5 different types of bread. If Hank makes a sandwich using 2 types of meat, one type of cheese, and 1 type of bread, how many different sandwiches could he make?

(A) 24 (B) 600 (C) 1,365 (D) 300 (E) 20
(F) 3,375 (G) 45 (H) 32,760 (J) 455
(K) none of the others

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14. (5 points) Is the sentence $(A \cup B)' = A' \cup B'$ always true, always false, or sometimes true and sometimes false?

(A) always true (B) sometimes true and sometimes false (C) always false
(D) none of the others

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15. (5 points) 6 assistants are to be assigned to administer exams in 3 different rooms: AH 100, BH 120, and CH 005. If 2 assistants are to be assigned to each room, in how many different ways can this be done?

(A) 216 (B) 21 (C) 720 (D) 18 (E) 729
(F) 22 (G) 90 (H) 3,375 (J) 45 (K) none of the others

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16. (5 points) According to the rules of a game, if one lands in the dungeon, then to escape one must roll a die until (i) the sum of the numbers rolled is at least 5 or (ii) some number has been rolled twice. If one keeps a record of the numbers that are rolled as they are rolled, how many different ways are there to escape from the dungeon.

(A) 51 (B) 58 (C) 70 (D) 31 (E) 46
(F) 44 (G) 52 (H) 30 (J) 56 (K) 61
(L) 60 (M) 57 (N) 62 (P) 66 (Q) 67
(R) 45 (S) 71 (T) 41 (U) 36 (V) 37
(X) 47 (Y) 50 (Z) none of the others

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17. (5 points) A seminar class contains 7 seniors, 4 juniors, and 2 sophomores. If 4 of these students are selected at random for a project, what is the probability that exactly 2 of them are juniors?

(A)	63/143	(B)	36/715	(C)	216/715	(D)	2/13	(E)	42/715
(F)	38/17,160	(G)	1/78	(H)	none of the others				

18. (5 points) The linen closet has 7 different white towels, 3 different brown towels, and 4 different yellow towels. If Sam takes out 2 towels at random for his daughter to take to summer camp, what is the probability that they are different colors?

(A)	84/182	(B)	30/91	(C)	14/91	(D)	61/91	(E)	1/2
(F)	84/91	(G)	61/182	(H)	77/91	(J)	14/182		
(K)	none of the others								

19. (5 points) A bucket of tennis balls contains 7 white balls, 3 yellow balls, and 2 orange balls. Joe takes one ball at random from the bucket and serves it. Then he takes another ball at random from the bucket and serves it. What is the probability that either the first ball is yellow and the second ball is white or both balls are orange?

(A)	23/132	(B)	84/144	(C)	22/66	(D)	21/66
(E)	26/132	(F)	25/144	(G)	42/132	(H)	none of the others

20. (5 points) A sale table has 6 different white shirts, 4 different yellow shirts, and 3 different blue shirts. If Anne buys 4 of these shirts for her husband, how many different possible outcomes have at least 1 blue shirt and at least 1 white shirt?

(A)	990	(B)	11,304	(C)	11,256	(D)	700
(E)	471	(F)	16,800	(G)	126	(H)	111
(J)	469	(K)	1,980	(L)	none of the others		

EXAM II

Multiple Choice (5 points each)

1. An unfair die is rolled once. The probabilities are as follows:

$$Pr[\{1\}] = Pr[\{2\}] = \ldots = Pr[\{5\}] = \frac{1}{10}, \text{ and } Pr[\{6\}] = \frac{1}{2}.$$

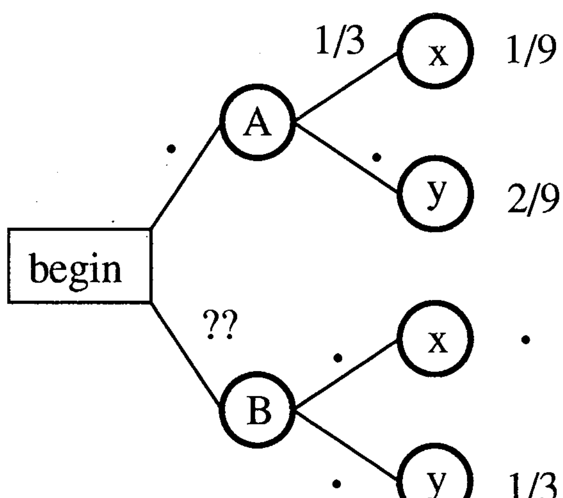
Let E be the event that the outcome is even. What is $Pr[\text{E}]$?

(a) $\frac{7}{10}$ (b) $\frac{1}{2}$ (c) $\frac{5}{16}$ (d) $\frac{1}{3}$ (e) none of the others

2. A sample space has outcomes σ_1, σ_2, and σ_3. σ_1 is four times as likely as σ_3, σ_2 is three times as likely as σ_3. What is $Pr[\{\sigma_2\}]$?

(a) $\frac{1}{3}$ (b) $\frac{3}{7}$ (c) $\frac{3}{8}$ (d) $\frac{7}{8}$ (e) none of the others

3. Find the missing number at ?? in the tree:



(a) $\frac{5}{6}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$ (e) none of the others

4. I have change in my pocket with probability 0.3 and have bills in my wallet with probability 0.2. The probability that I have *both* change and bills is 0.01. What is the probability that I have no money (neither change nor bills)?

(a) 0.4 (b) 0.5 (c) 0.6 (d) 0.7 (e) none of the others

5. There are twelve "face cards" in a standard 52-card deck (the Jack, Queen, and King in each of the four suits). A card is drawn at random from a standard deck. What is the probability that it is *not* a face card given that it is not a King?

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{5}{6}$ (e) none of the others

6. Events A and B are independent. $Pr[\mathrm{A}] = \frac{1}{3}$, and $Pr[\mathrm{B}] = \frac{1}{4}$. Find $Pr[(\mathrm{A} \cup \mathrm{B})']$.

(a) $\frac{1}{2}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) $\frac{11}{12}$ (e) none of the others

7. Any particular day in the life of history's happiest man will be a bad day with probability $\frac{1}{1000}$. He will live exactly 36,500 days. Find the expected number of bad days in his life.

(a) $\frac{73}{2}$ (b) $\frac{2}{73}$ (c) $\frac{1}{73}$ (d) 73 (e) none of the others

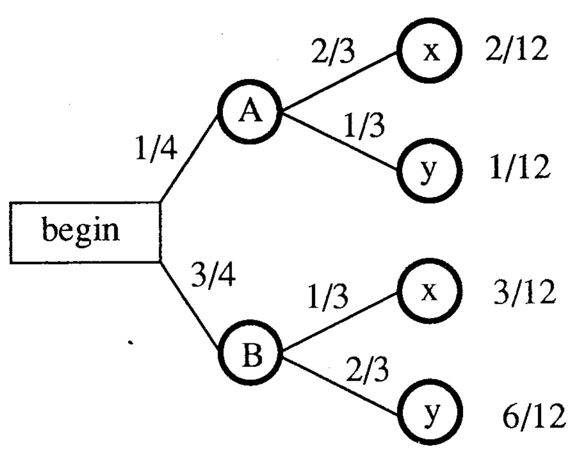
8. A random variable has the probability density function shown in the table below.

x_j	p_j
1	1/10
2	2/10
3	3/10
4	4/10

What is $E[X]$ for this function?

(a) 2 (b) 2.5 (c) 3 (d) 3.5 (e) none of the others

9. What is $Pr[\mathrm{A}|\mathrm{x}]$ for the tree below?



(a) $\frac{1}{4}$ (b) $\frac{2}{5}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$ (e) none of the others

10. Some bored cashiers notice that 80% of their customers order fish and 20% do not. Of those ordering fish, 60% also have fries. Of those not ordering fish, 40% have fries. What is the probability that a random customer orders fries?

(a) 48% (b) 50% (c) 56% (d) 58% (e) none of the others

11. Two computer programs are written in BASIC programming language and two others are in Lisp. Two of the four programs are run at random. Let X denote the number of BASIC programs run. Which of the following is the Probability Density Function for X?

(a)

x_j	p_j
0	1/6
1	2/3
2	1/6

(b)

x_j	p_j
0	1/3
1	1/3
3	1/3

(c)

x_j	p_j
0	1/4
1	1/2
2	1/4

(d)

x_j	p_j
0	0/3
1	1/3
2	2/3

(e) none of the others

12. A student has four multiple choice questions left on an exam, each with five possible answers. He's out of time so he selects answers at random. What is the probability that at most three answers are correct?

(a) $\frac{4}{5}$ (b) $\frac{19}{20}$ (c) $\frac{255}{256}$ (d) $\frac{624}{625}$ (e) none of the others

13. A fair die is rolled three times. What is the probability that a "5" appears exactly twice?

(a) $C(6,1)\cdot\left(\frac{2}{3}\right)^2\cdot\left(\frac{1}{3}\right)^2$ (b) $C(3,2)\cdot\left(\frac{5}{6}\right)^2\cdot\left(\frac{1}{6}\right)^1$ (c) $C(6,1)\cdot\left(\frac{2}{3}\right)^1\cdot\left(\frac{1}{3}\right)^2$
(d) $C(3,2)\cdot\left(\frac{1}{6}\right)^2\cdot\left(\frac{5}{6}\right)^1$ (e) none of the others

14. Events E and F are independent in a sample space S, $Pr[\text{E}] = .3$ and $Pr[\text{F}] = .6$. Find $Pr[\text{E}|\text{F}']$.

(a) $\frac{1}{4}$ (b) .5 (c) .3 (d) $\frac{3}{4}$ (e) none of the others

15. A car is selected at random from among a Chevrolet, a Ford, and a Honda. The Chevrolet is known to start with probability $\frac{6}{10}$, the Ford with probability $\frac{5}{10}$, and the Honda with probability $\frac{7}{10}$. If the car selected starts, find the probability that it is *not* the Ford.

(a) $\frac{13}{30}$ (b) $\frac{25}{30}$ (c) $\frac{13}{18}$ (d) $\frac{5}{18}$ (e) none of the others

True or False (2 points each)

_________ 1. A fair die is rolled twice. Let A be the event that the first roll is even. Let B be the event that the *sum* of the rolls is even. Then A and B are independent events.

_________ 2. For any event E, we have $Pr[\mathrm{E}] = \frac{1}{\mathrm{Pr}[\mathrm{E}']}$.

_________ 3. A random variable cannot take negative values.

_________ 4. A Bernoulli trial has only two possible outcomes.

_________ 5. The conditional probability of A given B is always at most the probability of A (in other words, for all events A and B we have $Pr[\mathrm{A}|\mathrm{B}] \leq \mathrm{Pr}[\mathrm{A}]$).

MIDTERM – VERSION 1

1. According to a poll, 29 % of voters support budget reform (i.e., a balanced budget amendment), 52 % of voters support health care reform, and 23 % of voters oppose both reforms. What is the probability that a randomly selected voter supports exactly one of these reforms?

(A)	0.00	(B)	0.77	(C)	0.78	(D)	0.73	(E)	0.23
(F)	0.04	(G)	0.25	(H)	0.12	(J)	0.48	(K)	0.81
(L)	none of the others								

2. A real estate company has 12 female agents and 8 male agents. If each of 400 customers is randomly assigned to one of these agents, what is the expected number who would be assigned to female agents?

(A)	133	(B)	200	(C)	240	(D)	100	(E)	260
(F)	300	(G)	160	(H)	120	(J)	250	(K)	none of the others

3. 70 % of a local catalogue company's customers live in Indiana. Of its Indiana customers, 30 % live in large cities, 20 % live in small towns, and 50 % live in rural areas. Of its non-Indiana customers, 10 % live in large cities, 30 % live in small towns, and 60 % live in rural areas. If a salesperson selects a customer at random, what is the probability that this customer lives in a large city or a small town?

(A)	0.90	(B)	0.43	(C)	0.00	(D)	0.45	(E)	0.76
(F)	0.53	(G)	0.47	(H)	0.77	(J)	none of the others		

4. A store has the following videos to rent: one copy of *Basic Instinct*, one copy of *Fatal Attraction*, one copy of *Dave*, one copy of *Age of Innocence*, one copy of *Casablanca*, one copy of *Carlucci's Way*, and one copy of *Twelfth Night*. If Mary, Sam, and Bill each rent one of these videos for tonight, how many different outcomes are possible?

(A)	5,040	(B)	120	(C)	1	(D)	35	(E)	27
(F)	21	(G)	3	(H)	2,187	(J)	210	(K)	343
(L)	7	(M)	18	(N)	6	(P)	none of the others		

5. According to the description of a new lottery game, the probability of winning \$ 500 is 0.01; the probability of winning \$ 200 is 0.05; the probability of winning \$ 100 is 0.14; the probability of winning \$ 50 is 0.20; and the probability of winning \$ −10 (i.e., losing the purchase price of the ticket) is 0.60. (This lottery refunds the purchase price of the ticket when one wins \$ 500, \$ 200, \$ 100, or \$ 50. So, for example, if one "wins \$ 500," then one really does win \$ 500.) What are the expected winnings (in dollars) for this game?

(A)	490.00	(B)	33.00	(C)	29.00	(D)	45.00	(E)	200.00
(F)	840.00	(G)	168.00	(H)	66.00	(J)	266.67	(K)	16.50
(L)	none of the others								

6. According to a marketing survey of 1,080 people, 150 of them shop at Ayres, 155 shop at Lazarus, 180 shop at Target, 35 shop at Ayres and Lazarus, 45 shop at Ayres and Target, 40 shop at Lazarus and Target, and 15 shop at all three of these stores. How many of those surveyed shop at none of these stores?

(A)	745	(B)	730	(C)	460	(D)	720	(E)	700
(F)	505	(G)	685	(H)	675	(J)	none of the others		

7. Every car in the Library's parking lot has an A parking permit, a C parking permit, or an E parking permit (no car has two or more different permits). 40 % of the cars have A parking permits, 10 % of the cars have C parking permits, and 50 % of the cars have E parking permits. Also, 80 % of the cars with A parking permits are legally parked, 70 % of the cars with C parking permits are legally parked, and 60 % of the cars with E parking permits are legally parked. If a traffic patrol person checks a randomly selected car in the Library's parking lot, what is the probability that it is *not* legally parked?

(A)	0.500	(B)	0.700	(C)	0.200	(D)	0.300
(E)	0.400	(F)	0.310	(G)	0.333	(H)	0.690
(J)	0.900	(K)	none of the others				

8. 6 sopranos, 3 tenors, and 4 basses perform in an opera competition. Each of the judges John, Elizabeth, and Thomas nominates one of these performers for the first prize. The moderator records who nominates which performer. How many different outcomes are possible?

(A)	1,716	(B)	2,197	(C)	1,594,323	(D)	39	(E)	13
(F)	25	(G)	80	(H)	286	(J)	150	(K)	216
(L)	72	(M)	17,280	(N)	none of the others				

9. 70 % of the students in a class live in McAcorn dorm and the rest live in Driscoe dorm. 90 % of those who live in McAcorn like their rooms, and 20 % of those who live in Driscoe like their rooms. If a student selected at random does not like her room, what is the probability that she lives in McAcorn?

(A)	$\frac{2}{23}$	(B)	$\frac{31}{100}$	(C)	$\frac{6}{13}$	(D)	$\frac{9}{11}$	(E)	$\frac{21}{23}$
(F)	$\frac{1}{9}$	(G)	$\frac{24}{31}$	(H)	$\frac{7}{31}$	(J)	$\frac{69}{100}$	(K)	$\frac{7}{13}$
(L)	$\frac{1}{3}$	(M)	none of the others						

10. A list of potential customers contains the names and telephone numbers of 2 Bloomington residents, 7 Indianapolis residents, and 2 South Bend residents. A computer randomly selects 3 different people from this list for a salesperson to call. What is the probability that 1 resident from each city is selected?

(A)	$\frac{28}{165}$	(B)	$\frac{1}{28}$	(C)	$\frac{1}{90}$	(D)	$\frac{28}{1,331}$	(E)	$\frac{1}{27}$
(F)	$\frac{1}{3}$	(G)	$\frac{11}{28}$	(H)	$\frac{3}{11}$	(J)	$\frac{8}{7}$	(K)	$\frac{1}{15}$
(L)	$\frac{14}{495}$	(M)	1	(N)	none of the others				

11. According to a survey, 30 % of the population own an imported automobile and 80 % own an imported television. If owning an imported automobile and owning an imported television were independent events, what would be the probability that a person selected at random would own neither an imported automobile nor an imported television?

(A) 0.86 (B) 0.24 (C) 0.14 (D) 0.62 (E) 0.76
(F) 0.70 (G) 0.80 (H) 0.30 (J) 0.90 (K) 0.20
(L) none of the others

---

12. An emergency medical team has 5 men and 3 women. 3 of them are selected at random to respond to an emergency call. A random variable X is defined to be the number of men selected. Find $Pr[X = 1]$?

(A) $\frac{1}{56}$ (B) $\frac{3}{56}$ (C) $\frac{75}{512}$ (D) $\frac{45}{512}$ (E) $\frac{15}{28}$
(F) $\frac{1}{7}$ (G) $\frac{11}{336}$ (H) $\frac{15}{56}$ (J) $\frac{11}{512}$ (K) $\frac{5}{28}$
(L) $\frac{11}{24}$ (M) $\frac{5}{56}$ (N) $\frac{135}{512}$ (P) none of the others

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13. In a group of students waiting for an advisor, 40 % want to take a psychology course, 30 % want to take a history course, and 11 % want to take both a psychology course and a history course. If an advisor selects one of these students at random for her next advising appointment and this student does *not* want to take a history course, what is the probability that the student wants to take a psychology course?

(A) $\frac{40}{59}$ (B) $\frac{29}{100}$ (C) $\frac{19}{30}$ (D) $\frac{19}{60}$ (E) $\frac{29}{70}$
(F) $\frac{29}{40}$ (G) $\frac{30}{41}$ (H) $\frac{30}{49}$ (J) $\frac{2}{5}$ (K) $\frac{41}{70}$
(L) $\frac{3}{10}$ (M) $\frac{11}{51}$ (N) $\frac{11}{100}$ (P) $\frac{19}{100}$ (Q) $\frac{40}{51}$
(R) $\frac{11}{40}$ (S) none of the others

---

14. A mutual fund company offers its clients 6 different stock funds, 4 different bond funds, and 3 different money market funds. If a novice investor chooses 3 of the stock funds, 2 of the bond funds, and 2 of the money market funds for his investments, how many different outcomes are possible?

(A) 8,640 (B) 138 (C) 32 (D) 360 (E) 241
(F) 7 (G) 864 (H) 13 (J) 29 (K) 72
(L) 31,104 (M) 12 (N) none of the others

---

15. 30 % of the shoes in a shoe store are imported. 90 % of the imported shoes have synthetic soles, and 20 % of the domestic (i.e., nonimported) shoes have synthetic soles. What is the probability that a shoe that has a synthetic sole is domestic? (Assume the shoe was selected at random in the store.)

(A) $\frac{3}{100}$ (B) $\frac{56}{59}$ (C) $\frac{27}{41}$ (D) $\frac{1}{5}$ (E) $\frac{27}{100}$
(F) $\frac{14}{41}$ (G) $\frac{3}{59}$ (H) $\frac{14}{25}$ (J) $\frac{7}{50}$ (K) $\frac{41}{100}$
(L) none of the others

---

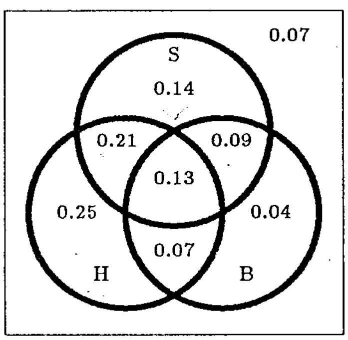
16. In an industrial psychology experiment, subjects are asked to select randomly 1 of 3 buttons on a panel and push it. 2 of the buttons are round and 1 of the buttons is square. If 5 subjects each perform the experiment independently of one another and actually do select a button at random, what is the probability that exactly 2 of them would push a square button?

(A) $\frac{2}{3}$	(B) $\frac{1}{3}$	(C) $\frac{2}{5}$	(D) $\frac{8}{243}$	(E) $\frac{2}{15}$
(F) $\frac{1}{9}$	(G) $\frac{80}{243}$	(H) $\frac{4}{243}$	(J) $\frac{83}{243}$	(K) $\frac{40}{243}$
(L) $\frac{26}{81}$	(M) none of the others			

17. 4 Canadian companies, 6 U.S. companies, 5 British companies, and 3 French companies have submitted bids to provide international investment advice to a wealthy family. If 2 of these companies are selected at random, what is the probability that 1 will be a North American company and 1 will be a European company?

(A) $\frac{1}{80}$	(B) $\frac{80}{153}$	(C) 1	(D) $\frac{2}{17}$	(E) $\frac{40}{153}$
(F) $\frac{20}{81}$	(G) $\frac{1}{18}$	(H) $\frac{1}{17}$	(J) $\frac{9}{40}$	(K) $\frac{1}{2}$
(L) none of the others				

18. A network surveyed viewers to determine the likelihood that they like figure skating (S), ice hockey (H), or bobsled racing (B). The results are presented in the Venn diagram below. On the basis of this diagram, what is the probability that a randomly selected viewer likes exactly one of figure skating and ice hockey or does not like bobsled racing?



(A) 0.46	(B) 0.76	(C) 0.21
(D) 0.34	(E) 1.00	(F) 0.28
(G) 0.96	(H) 0.60	(J) 0.83
(K) 0.89	(L) 0.39	(M) 0.41
(N) 0.67	(P) 0.00	(Q) none of the others

(The circle labelled "S" represents those who like figure skating; the circle labelled "H" represents those who like ice hockey; and the circle labelled "B" represents those who like bobsled racing. The number in each region of the diagram is the fraction of the membership in that alternative, as determined by the survey. Thus, for example, the probability that a viewer selected at random likes only figure skating is 0.14.)

19. A survey of the residents of one county showed that 90 % of them favor gun control laws. Also, 60 % of those who favor gun control laws do *not* own guns, and 70 % of those who do *not* favor gun control laws do own guns. If a resident of this county selected at random owns a gun, what is the probability that this person favors gun control laws?

(A)	$\frac{18}{19}$	(B)	$\frac{54}{61}$	(C)	$\frac{1}{19}$	(D)	$\frac{9}{25}$	(E)	$\frac{2}{3}$
(F)	$\frac{3}{10}$	(G)	$\frac{36}{43}$	(H)	$\frac{3}{5}$	(J)	$\frac{2}{5}$	(K)	$\frac{27}{50}$
(L)	$\frac{1}{13}$	(M)	$\frac{12}{13}$	(N)	$\frac{3}{100}$	(P)	$\frac{7}{43}$	(Q)	$\frac{7}{100}$
(R)	$\frac{1}{3}$	(S)	none of the others						

20. A store, as part of a promotional raffle, is offering each raffle winner a choice of a light yellow T-shirt, a dark blue T-shirt, a dark green T-shirt, and a dark purple T-shirt. If each of 4 winners randomly and independently selects one of these options, what is the probability that at most 3 winners select a dark colored T-shirt?

(A)	$\frac{175}{256}$	(B)	$\frac{3}{64}$	(C)	$\frac{61}{64}$	(D)	$\frac{3}{256}$	(E)	$\frac{255}{256}$
(F)	$\frac{27}{64}$	(G)	$\frac{1}{4}$	(H)	$\frac{37}{64}$	(J)	$\frac{27}{256}$	(K)	$\frac{3}{4}$
(L)	none of the others								

21. A winter sports team consists of 4 male skaters, 6 female skaters, 7 male skiers, and 5 female skiers (no one competes in both skating and skiing). If 2 team members are selected at random to lead the team in the opening parade, what is the probability that 1 is male and 1 is female given that no male skiers are selected?

(A)	$\frac{105}{121}$	(B)	$\frac{1}{15}$	(C)	$\frac{10}{11}$	(D)	$\frac{44}{105}$	(E)	$\frac{2}{21}$
(F)	$\frac{44}{225}$	(G)	$\frac{105}{242}$	(H)	$\frac{1}{44}$	(J)	$\frac{4}{21}$	(K)	$\frac{22}{105}$
(L)	$\frac{1}{11}$	(M)	$\frac{1}{225}$	(N)	none of the others				

22. A work shift has 7 machinists and 5 electricians. If 4 of these workers are selected at random for a union's grievance board, how many different possible boards would have at least 2 electricians?

(A)	114	(B)	210	(C)	420	(D)	10	(E)	250
(F)	285	(G)	20	(H)	6	(J)	9,990	(K)	84
(L)	495	(M)	none of the others						

23. A medical equipment salesperson has the following assignment:

> "Here is a list of 3 hospitals — A, B, and C. Go to each one in that order and try to sell it an MRI machine or a CAT scan machine. You should stop for the day when and only when you have sold 1 MRI machine or 2 CAT scan machines or have gone to all three hospitals."

Assume that each hospital buys at most 1 machine and that the salesperson stops as soon as permitted. The salesperson keeps a record of what happens at hospital A (i.e., *no sale* or *only 1 MRI machine sold* or *only 1 CAT scan machine sold*), then what happens at hospital B if she visits it, and then what happens at hospital C if she visits it. How many different outcomes are possible?

(A)	10	(B)	13	(C)	27	(D)	19	(E)	21
(F)	18	(G)	12	(H)	7	(J)	15	(K)	11
(L)	16	(M)	24	(N)	9	(P)	6	(Q)	14
(R)	20	(S)	3	(T)	8	(U)	36	(V)	none of the others

MIDTERM – VERSION 2

1. Jake's Used Car lot has 4 Mercedes (all different colors) and 5 Infinities (all different colors). How many ways can Jake line them up for a picture, if he keeps all Mercedes together and all Infinities together?

A) 9! B) 120 C) 5760 D) 2880

E) 240 F) none of the others

2. Using the setting of problem 1, suppose two cars are selected at random (without replacement) from Jake's lot. What is the probability that they are both Infinities, given that they are not both Mercedes?

A) 4/9 B) 1/3 C) 2/3 D) 5/9

E) 2/9 F) none of the others

3. Roberta Nite (Bobby to her friends) has 9 players on her chess team. How many ways can she select a subset of 5 to compete in an upcoming match?

A) 9x8x7x6x5 B) 3024 C) 84 D) 126

E) 72 F) none of the others

4. Suppose that Roberta Nite's 9-member chess team includes R. Fisher and J. Polgar. How many ways can Roberta select 5 members from the team, if she decides to include either Fisher or Polgar, but not both?

A) 84 B) 126 C) 35 D) 210

E) 70 F) none of the others

5. Suppose that two fair dice are rolled and the **product** is noted. What is the probability that the product is either 20, 24, or 25?

A) 3/36 B) 2/9 C) 5/36 D) 4/36

E) 1/6 F) none of the others

6. A modified deck of cards contains 4 aces, 3 kings, 2 queens, and one jack, for a total of 10 cards. If two cards are selected at random (without replacement), what is the probability that at least one is an ace?

A) $\frac{2}{3}$ B) C(10,2)/P(10,2) C) $\frac{2}{15}$ D) $\frac{24}{25}$

E) $\frac{1}{3}$ F) none of the others

7. Using the same deck of cards as above, and again selecting 2 cards at random, what is the probability that neither card is a jack?

A) $\frac{1}{3}$ B) $\frac{2}{15}$ C) $\frac{4}{5}$ D) $\frac{1}{5}$

E) $\frac{2}{3}$ F) none of the others

8. Suppose that E and F are events in a sample space S, with $Pr[E] = .5$, $Pr[F] = .7$ and $Pr[E' \cap F] = .3$. What is $Pr[E' \cap F']$?

A) .1 B) .5 C) .3 D) .2

E) .4 F) none of the others

9. Suppose E, F, and G are events in a sample space S. Suppose also that we know the following probabilities: $Pr[E] = .4$, $Pr[F] = .5$, $Pr[G] = .6$, $Pr[E \cap F] = .2$, $Pr[E \cap G] = .15$, $Pr[F \cap G] = .25$, and $Pr[E \cap F \cap G] = .05$. What is $Pr[E \cup F \cup G]$?

A) .95 B) .75 C) .05 D) .90

E) 1.0 F) none of the others

10. In the setting of problem 9 above, which of the pairs of events {E,F}, {E,G}, {F,G} are independent?

A) {F , G} only B) All three pairs C) {E , G} only D) {E , F} only

E) {E , G} and {E , F} F) none of the others

11. In the new IU Lottery, you pay $1 to play, and then you pick an **ordered** pair of numbers (repetitions allowed) from the set {0,1,2,3,4,5,6,7,8,9}. Thus, (2,5) and (4,4) would be legal picks. Next, an **ordered** pair of numbers is randomly selected (*with replacement*) from this set, and if the pair selected exactly matches yours, then you get $50. If the pair does not match yours, you get nothing? What is your expected **net return** (the amount you receive minus the amount you paid)?

A) -.75 B) +.50 C) -.25 D) -.50

E) +49 F) none of the others

12. The table shown below gives the probability density for the random variable X. Find the missing value, denoted by ? and the expected value of X.

Values of X	Probabilities
-1	.2
0	.3
2	?
4	.4

A) ? = .4, E[X] = .16 B) ? = .2, E[X] = .16 C) ? = .1, E[X] = 0 D) ? = .1, E[X] = .16

E) ? = .1, E[X] = 1.6 F) none of the others

13. Sam orders track shoes for his team from both West Bay and Frog Sports shoes. He gets 30% from West Bay and the rest from Frog. Over the years, he has noticed that 10% of the shoes from West Bay are mis-sized and 5% of those from Frog are mis-sized. What is the probability that a randomly selected shoe will be mis-sized?

A) .65 B) .35 C) .065 D) .035

E) .03 F) none of the others

14. Using the setting of problem 13, suppose that a randomly selected shoe is found to be of the correct size. What is the probability that it came from West Bay?

A) 30/65 B) 270/935 C) .27 D) .935

E) .3 F) none of the others

15. An urn contains 3 red balls and 2 blue balls. An experiment consists of repeatedly drawing a ball at random, noting the color, and then **returning** the ball to the urn. If five balls are drawn in all, what is the probability that four are red and one is blue?

A) 81/125 B) 1/625 C) C(5,4)C(5,1)/C(5,5) D) 81/625

E) 162/625 F) none of the others

16. Using the experiment of problem 15, suppose 125 balls are drawn in all. What is the expected number of blue balls drawn?

A) 62.5 B) 55 C) 75 D) 50

E) 37.5 F) none of the others

17. If K and L are subsets of U with $n(K \cup L) = 100$ and $n(L) = 100$, which of the following **must be true**?

A) $L = U$ B) $L \subset K$ C) $K = \phi$ D) $K \cap L = \phi$

E) $K \subset L$ F) none of the others

18. Let U = {1,2,3,4,5,6,7,8,9} be a universal set with subsets E = {2,4,6,8}, O = {1,3,5,7,9}, L= {1,2,3}, M = {4,5,6} , and H = {7,8,9}. Which of the following is the set $(L \cup H') \cap (E \cup M)'$?

A) {1,3,7,9} B) {1,2,3,4,5,6} C) L D) {1,3}

E) {2,4,6} F) none of the others

19. A sack contains 2 red apples, 4 yellow apples, and 1 green apple. If 3 apples are selected at random (without replacement), what is the probability that 2 are red and 1 is green?

A) 2/5 B) C(7,3) C) 1/210 D) 2/35

E) 1/35 F) none of the others

20. Using the experiment of problem 19 above, what is the probability that **at least** two of the apples selected are yellow?

A) 22/35 B) 1/35 C) 3/5 D) 4/35

E) 18/35 F) none of the others

21. Let K and L be subsets of a universal set U, and suppose $n(K' \cap L) = n(K \cap L') = 20$, $n(K' \cap L') = 25$, and $n(U) = 100$. Find $n(K' \cup L')$.

A) 65 B) 80 C) 35 D) 55

E) 45 F) none of the others

22. An experiment consists of repeatedly drawing a ball from an urn which contains 3 red balls and 5 blue balls. After each drawing, the color of the ball is noted and that ball is discarded. The experiment continues until two **consecutive** red balls are drawn or two **consecutive** blue balls are drawn. If an outcome consists of an ordered list of the colors of the balls drawn, how many outcomes exist?

A) 11 B) 13 C) 9 D) 10

E)12 F) none of the others

23. Suppose that 200 students in a Finite Mathematics class are asked about their TV viewing habits. 100 students say they regularly watch Letterman, 70 regularly watch the Simpsons, and 80 regularly watch ESPN's early sports show. Also, 30 watch both the Simpsons and Letterman, and 40 watch both ESPN early sports and Letterman. Among all the students in this class, 20 regularly watch all three of these programs. It is also known that 30 of those who watch the Simpsons do not watch anything else. How many watch ESPN's early sports, but not Letterman?

A) 20 B) 40 C) not enough information D) 30

E) 50 F) none of the others

24. Peter and Mary decide to tour without Paul, and they decide to visit 5 world-class cities selected from the set {Chicago, New York, London, Paris, Bloomington, Los Angeles, San Fransisco, Mexico City}. If a tour is an ordered list of 5 of these cities, how many tours exist with Bloomington first and Paris last?

A) P(3,3) B) C(3,3) C) 5! D) C(6,3)

E) P(6,3) F) none of the others

25. A carefully constructed die has the property that the number 1 comes up twice as often as the number 2, the number 2 comes up twice as often as the number 3, the number 6 comes up as often as 1, the number 5 comes up as often as 2, and the number 4 comes up as often as 3. For this die, what is Pr[3]?

A) 5/14 B) 4/14 C) 1/6 D) 1/14

E) 1/3 F) none of the others

EXAM III – VERSION 1

1. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 4 & 2 \end{pmatrix}$$

What is the entry in the second row and third column of A^{-1}?

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) -2 (d) 1 (e) none of the others

2. Find all solutions of the system of equations with the augmented matrix

$$\left(\begin{array}{ccc|c} 2 & 2 & 2 & 4 \\ 0 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 \end{array}\right).$$

(a) $x = 2, \quad y = 1, \quad z = 0$
(b) $x = 2, \quad y = 1, \quad z$ arbitrary
(c) $x = 2, \quad y = 1 - z, \quad z$ arbitrary
(d) $x = 2, \quad y = 0, \quad z = 0$
(e) none of the others

3. For which value of k does the matrix A *not* have an inverse?

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 2 & 4 \\ 3 & 4 & k \end{pmatrix}$$

(a) $k = 1$ (b) $k = 2$ (c) $k = 5$ (d) $k = 6$ (e) none of the others

4. Which of the statements given below accurately describes the set of solutions for the system of equations with augmented matrix

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 2 \end{array}\right).$$

(a) the system has a unique solution
(b) the system has no solution
(c) the system has an infinite number of solutions and for all solutions $x = 2$
(d) the system has an infinite number of solutions and for all solutions $z = 0$
(e) none of the others

5. Tom and Dick have a fruit juice stand at which they make both a sweet drink and a tart drink. Each glass of sweet drink uses .8 of the juice of an orange and .2 of the juice of a lemon. Each glass of tart drink uses .6 of the juice of an orange and .7 of the juice of a lemon. If they have 50 oranges and 40 lemons, how many glasses of each type of drink should they make to use all of the oranges and lemons?
(a) 20 sweet, 40 tart (b)25 sweet, 50 tart (c) 15 sweet, 30 tart
(d) 30 sweet, 60 tart (e) none of the others

6. Which of the following is *not* a corner point of the feasible set given by $x \geq 0$, $y \geq 0$, $2x + y \leq 4$, $x + 2y \leq 4$, $2x + 2y \leq 5$.

(a) (2,0) (b) (0,2) (c) $(1,\frac{3}{2})$ (d) $(\frac{4}{3},\frac{4}{3})$ (e) none of the others

7. Find the maximum of $2x - 3y$ on the feasible set given by $x \geq 0$, $2x + 3y \leq 5$, $2x - y \leq 5$.

(a) 0 (b) -5 (c) 5 (d) no maximum exists (e) none of the others

8. Find the minimum of $-x + 3y$ on the feasible set given by $x \geq 0$, $y \geq 0$, $x + y \geq 3$, $x + 2y \geq 4$.

(a) -4 (b) -9 (c) no minimum exists (d) 1 (e) none of the others

9. Let

$$A = \begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix} \quad B = \begin{pmatrix} 4 & -2 & 3 \\ 0 & 1 & -1 \end{pmatrix}.$$

What is the entry in row 2 and column 3 of $-2AB$?

(a) -26 (b) -13 (c) -20 (d) AB is not defined (e) none of the others

10. Which of the following matrices does *not* have an inverse?

(a) $\begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 2 \\ 1 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} -2 & -4 \\ 1 & 2 \end{pmatrix}$

(e) none of the others

11. Which of the following lines has slope 2?

(a) $x = 2$ (b) $y = 2$ (c) $x = 2y + 4$ (d) $2x - y = 4$ (e) none of the others

12. Which of the following lines is parallel to the line through the points (1,3) and (2,6)?

(a) $3x - y = 4$ (b) $x - 3y = 4$ (c) $y = 3$ (d) $x = 3$ (e) none of the others

13. What is the value of y at the intersection of the lines $2x + 3y = 6$ and $-x + 4y = 2$?

(a) $\frac{2}{11}$ (b) $\frac{6}{5}$ (c) $\frac{1}{2}$ (d) $\frac{10}{11}$ (e) none of the others

14. Which of the following points is *not* in the feasible set given by the constraints $2x + y \geq 4$, $x \geq y$, $x + y \leq 5$.

(a) (3,1) (b) (3,-3) (c) (2,2) (d) (2,-2) (e) none of the others

15. Let A be 3×4, B be 4×3, C be 3×3 and D be 4×4. Which of the following matrix products is *not* defined?

(a) ABC (b) BAD (c) DBA (d) CAB (e) none of the others

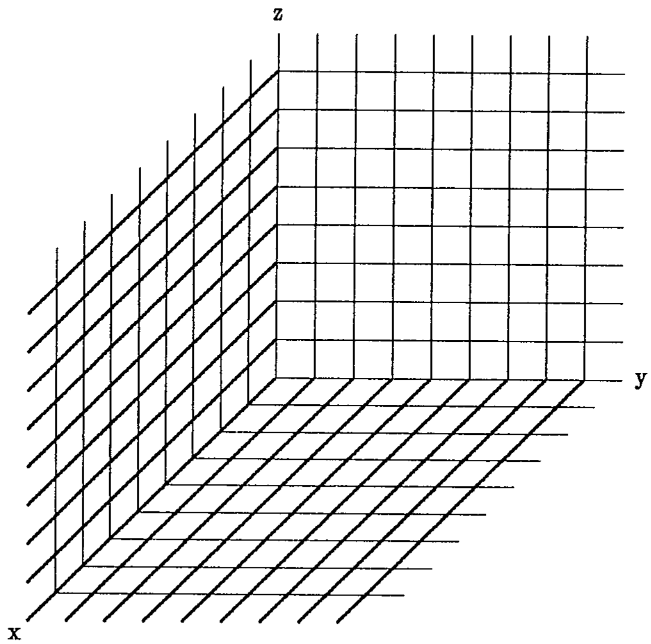
16. Use Gaussian elimination to find all solutions of the following system of equations. **Show reduced form!**

$$\begin{array}{rcrcrcrcr} x & - & 2y & & & + & 4w & = & -5 \\ x & - & 2y & + & z & + & w & = & 1 \\ 2x & - & 4y & + & z & + & 5w & = & -4 \\ 3x & - & 6y & + & 2z & + & 6w & = & -3 \end{array}$$

EXAM III – VERSION 2

1. (4 points) Find the equation of the line through the point $(4, 7)$ that is parallel to the line through the points $(-5, 10)$ and $(4, -17)$.

2. (4 points) Sketch the graph of the set of solutions of the equation $4x + 8y + 6z = 24$ on the Cartesian coordinate system below:



3. (4 points) Perform the following computation:

$$4\begin{bmatrix} -6 & 2 \\ -7 & -3 \\ 9 & 4 \end{bmatrix} - 5\begin{bmatrix} -8 & 8 \\ -2 & 7 \\ 3 & -6 \end{bmatrix} =$$

4. (4 points) Perform the following computation:

$$\begin{bmatrix} 7 & 2 & -5 \\ -5 & 0 & 9 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 5 & -8 \\ 0 & 4 \end{bmatrix} =$$

5. (5 points) Assume that A is a 2×3 matrix, B is a 3×2 matrix, C is a 3×3 matrix, and D is a 2×3 matrix. In each of the following parts, circle the word *Defined* if all of the operations are defined, and circle the word *Undefined* if any operation is undefined.

(a)	ACD	Defined	Undefined
(b)	AC	Defined	Undefined
(c)	CA	Defined	Undefined
(d)	$3B + 2C$	Defined	Undefined
(e)	A^2	Defined	Undefined

6. (8 points) In each of the following parts, circle *Yes* if the matrix is in reduced row echelon form, and circle *No* if the matrix is not in reduced row echelon form.

(a)	$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$	Yes	No
(b)	$\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 & 0 \end{bmatrix}$	Yes	No
(c)	$\begin{bmatrix} 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$	Yes	No
(d)	$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$	Yes	No

7. (8 points) Let A, B, and X be the following matrices:

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 2 & -1 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} -3 \\ -4 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

In each of the following parts, circle *Yes* if the column vector is a solution of the equation $AX = B$, and circle *No* if the column vector is not a solution of this equation.

(a) $\begin{bmatrix} 6 \\ 1 \\ -2 \end{bmatrix}$ Yes No (b) $\begin{bmatrix} -13 \\ 6 \\ 4 \end{bmatrix}$ Yes No

(c) $\begin{bmatrix} -8 \\ 2 \\ 2 \end{bmatrix}$ Yes No (d) $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ Yes No

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In each of the next four questions consider the system of linear equations that corresponds to the given augmented matrix and decide which of the following statements is true.

(A) The system has no solution.
(B) The system has exactly one solution.
(C) The system has exactly three solutions.
(D) The system has an infinite set of solutions with exactly one arbitrary parameter.
(E) The system has an infinite set of solutions with exactly two arbitrary parameters.
(F) The system has an infinite set of solutions with exactly three arbitrary parameters.
(G) None of the others is true.

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8. (4 points)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 4 \\ -1 & 1 & -1 & -3 \end{array}\right]$$

Answer: __________

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9. (4 points)

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

Answer: __________

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10. (4 points)

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 2 & 2 & 5 \\ -1 & 0 & -2 & -1 \end{array}\right]$$

Answer: __________

11. (4 points)

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ -1 & 1 & -2 & -1 \\ 2 & -2 & 4 & 2 \\ 1 & -1 & 2 & 1 \end{array}\right]$$

Answer: ____________

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12. (9 points) The Nectar Nutrition Company produces three types of liquid dietary supplements: Normal, High-Potency, and Lo-Cal. Producing 100 liters (l.) of Normal requires 16 liters of nectar A, 12 kilograms (kg.) of chemical B, 5 liters of preservative C, and 50 minutes for processing. Producing 100 liters of High-Potency requires 12 liters of nectar A, 3 kilograms of chemical B, 4 liters of preservative C, and 20 minutes for processing. Producing 100 liters of Lo-Cal requires 7 liters of nectar A, 9 kilograms of chemical B, 6 liters of preservative C, and 40 minutes for processing.
    (i) Formulate the process of determining the amounts of nectar A, chemical B, and preservative C, and time for processing required to produce specified quantities of the three types of dietary supplements as a matrix multiplication problem.
    (ii) Use part (i) to determine the amounts of nectar A, chemical B, and preservative C, and time for processing required to produce 600 liters of Normal, 400 liters of High-Potency, and 800 liters of Lo-Cal.

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13. (14 points) Use the Gauss-Jordan algorithm to find all solutions of the following system of linear equations:

$$\begin{aligned} x + y - 2z &= 9 \\ -2x - 2y + 6z &= -24 \\ 3x + 4y - 5z &= 25 \end{aligned}$$

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14. (14 points) Use the Gauss-Jordan algorithm to find all solutions of the following system of linear equations:

$$\begin{aligned} x_1 + 5x_2 - x_3 - 2x_4 &= -11 \\ -2x_1 - 10x_2 + 3x_3 + 5x_4 &= 27 \\ x_1 + 5x_2 - x_3 - x_4 &= -7 \end{aligned}$$

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15. (10 points) The Always Dry Roofing Company produces three grades of asphalt roofing shingles: A, B, and C (also known as 20-year, 25-year, and 30-year shingles, respectively). Producing 100 square feet (sq. ft.) of type A shingles requires 1 pound of fine gravel, 2 pounds of fiber glass, and 4 pounds of coal tar. Producing 100 square feet of type B shingles requires 2 pounds of fine gravel, 5 pounds of fiber glass, and 8 pounds of coal tar. Producing 100 square feet of type C shingles requires 3 pounds of fine gravel, 6 pounds of fiber glass, and 13 pounds of coal tar. The model for this process is

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 4 & 8 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},$$

where

x_1 = amount of type A shingles in units of 100 sq. ft.,
x_2 = amount of type B shingles in units of 100 sq. ft.,
x_3 = amount of type C shingles in units of 100 sq. ft.,
r_1 = amount of fine gravel in units of pounds,
r_2 = amount of fiber glass in units of pounds, and
r_3 = amount of coal tar in units of pounds.

If the company has 330 pounds of fine gravel, 740 pounds of fiber glass, and 1,360 pounds of coal tar available, how much of each type of shingle should the company produce to use all of these supplies? (Use techniques discussed in lectures.)

EXAM IV

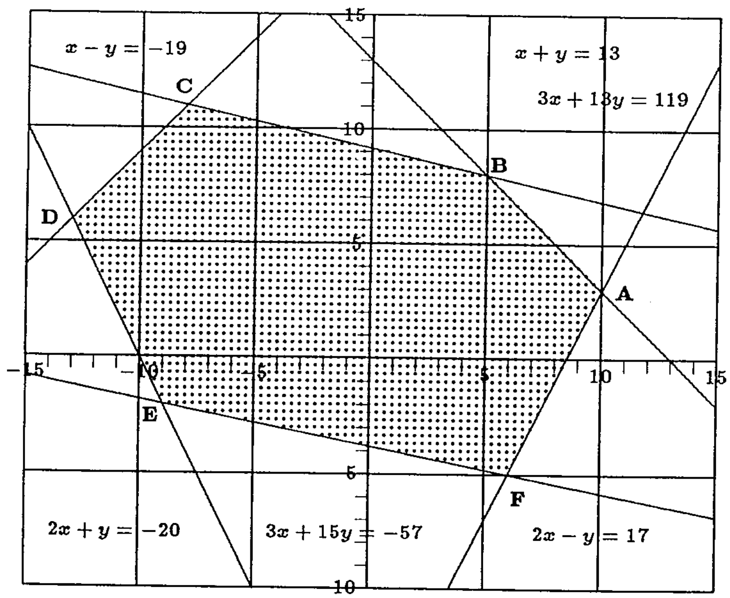
The graph of the set of points described by the inequalities

$$x + y \leq 13$$
$$3x + 13y \leq 119$$
$$x - y \geq -19$$
$$2x + y \geq -20$$
$$3x + 15y \geq -57$$
$$2x - y \leq 17$$

appears in Figure 1 to the right. The coordinates of the labelled corners of the set are as follows:

A: $(10, 3)$	B: $(5, 8)$
C: $(-8, 11)$	D: $(-13, 6)$
E: $(-9, -2)$	F: $(6, -5)$

Figure 1



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1. (5 points) (See Figure 1.) Find the minimum value of $z = -3x + y$ subject to the constraints

$$x + y \leq 13 \quad 3x + 13y \leq 119 \quad x - y \geq -19 \quad 2x + y \geq -20 \quad 3x + 15y \geq -57 \quad \text{and} \quad 2x - y \leq 17 .$$

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2. (5 points) (See Figure 1.) Find the maximum value of $z = 3x + 4y$ subject to the constraints

$$x + y \leq 13 \quad 3x + 13y \leq 119 \quad x - y \geq -19 \quad 2x + y \geq -20 \quad 3x + 15y \geq -57 \quad \text{and} \quad 2x - y \leq 17 .$$

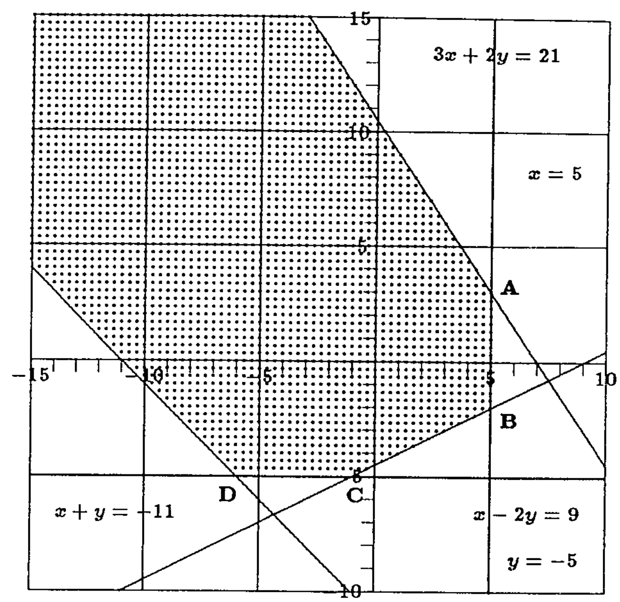
The graph of the set of points described by the inequalities

$$3x + 2y \leq 21$$
$$x \leq 5$$
$$x - 2y \leq 9$$
$$y \geq -5$$
$$x + y \geq -11$$

appears in Figure 2 to the right. The coordinates of the labelled corners of the set are as follows:

A: $(5, 3)$ B: $(5, -2)$
C: $(-1, -5)$ D: $(-6, -5)$

Figure 2



3. (6 points) (See Figure 2.) Find the maximum value of $z = -3x - 4y$ subject to the constraints

$$3x + 2y \leq 21 \quad x \leq 5 \quad x - 2y \leq 9 \quad y \geq -5 \quad \text{and} \quad x + y \geq -11 \quad .$$

4. (6 points) (See Figure 2.) Find the maximum value of $z = 4x + 3y$ subject to the constraints

$$3x + 2y \leq 21 \quad x \leq 5 \quad x - 2y \leq 9 \quad y \geq -5 \quad \text{and} \quad x + y \geq -11 \quad .$$

5. (6 points) (See Figure 2.) Find the minimum value of $z = -4x + 3y$ subject to the constraints

$$3x + 2y \leq 21 \quad x \leq 5 \quad x - 2y \leq 9 \quad y \geq -5 \quad \text{and} \quad x + y \geq -11 \quad .$$

6. (5 points) The inverse of the matrix $A = \begin{bmatrix} 7 & 3 & -2 \\ 4 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 3 & -1 \\ 0 & 1 & 2 \end{bmatrix}$. Use this information to solve the following system of linear equations:

$$7x + 3y - 2z = 25$$
$$4x + 2y - z = 15$$
$$-2x - y + z = -9$$

7. (8 points) Find the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -4 & -7 & -1 \\ 3 & 6 & 1 \end{bmatrix}.$$

8. (8 points) The technology for a certain open economy with goods G_1, and G_2 is $\begin{bmatrix} 0.6 & 0.2 \\ 0.1 & 0.7 \end{bmatrix}$.

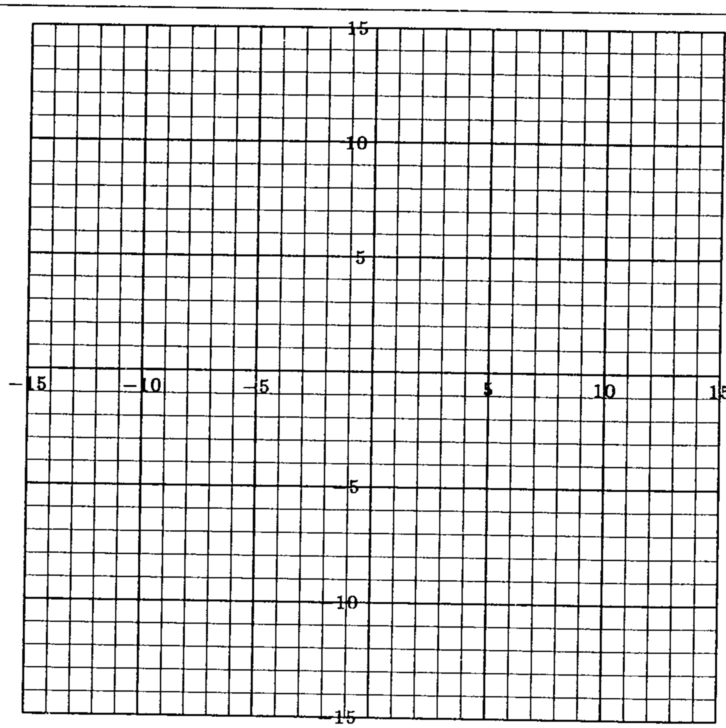
(a) Find the production schedule required to meet an external demand of $\begin{bmatrix} 40 \\ 120 \end{bmatrix}$ (in a steady-state).

(b) Find the production schedule required to meet an external demand of $\begin{bmatrix} 80 \\ 40 \end{bmatrix}$ (in a steady-state).

9. (8 points) A set of points is described by the inequalities

$$x \le 12$$
$$y \le 5$$
$$x \ge -5$$
$$y \ge -8$$
$$x + y \ge 2$$
$$-x + y \ge -6 \ .$$

Graph this set of points on the graph paper to the right and find the coördinates of all of its corner points.



10. (8 points) A Markov chain has the transition matrix

$$P = \begin{bmatrix} 2/7 & 5/7 \\ 6/7 & 1/7 \end{bmatrix} .$$

(a) Find the three-step transition matrix (i.e., $P(3)$) for this Markov chain.

(b) If this Markov chain begins in state 2, what is the probability that it will be in state 1 three observations later?

11. (8 points) A Markov chain has the transition matrix

$$P = \begin{bmatrix} 3/7 & 4/7 & 0 \\ 0 & 2/7 & 5/7 \\ 1/7 & 6/7 & 0 \end{bmatrix}$$

and an initial state matrix

$$X_0 = [2/3 \quad 1/3 \quad 0] \; .$$

(a) Find X_2.

(b) What is the probability that this Markov chain is in state 1 after two steps?

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12. (12 points) A Markov chain has the transition matrix

$$P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{8} & \frac{3}{4} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{8} & \frac{5}{8} \end{bmatrix} .$$

Use the procedure described in class to find a stable vector for this Markov chain.

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13. (3 points) In a certain country the technology of the aluminum mining industry, the aluminum smelting industry, and the electricity industry is such that (i) mining one ton of aluminum ore uses 0.001 tons of aluminum and 50 kilowatt hours of electricity, (ii) producing one ton of aluminum requires 4 tons of aluminum ore and 200 kilowatt hours of electricity, and (iii) producing 1 kilowatt hour of electricity requires 0.002 tons of aluminum and and 0.03 kilowatt hours of electricity. What is the technology matrix for this economy?

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14. (6 points) Joe watches every IU basketball game on television at home, and every game he orders a pepperoni pizza, a vegetarian pizza, or an inferno pizza (he orders only one pizza for each game). If he orders a pepperoni pizza for one game, then for the next game the probability that he will order a vegetarian pizza is 3/8 and the probability that he will order an inferno pizza is 5/8. If he orders a vegetarian pizza for one game, then for the next game he will order either a pepperoni pizza or an inferno pizza, and he is five times more likely to order an inferno pizza than a pepperoni pizza. If he orders an inferno pizza for one game, then for the next game he is equally likely to order a vegetarian pizza as an inferno pizza, and he is three times more likely to order a pepperoni pizza than an inferno pizza. View this situation as a Markov chain and construct its transition diagram and transition matrix.

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15. (6 points) **Formulate the following problem mathematically; DO NOT ATTEMPT TO SOLVE IT.** The XXX Paper Company produces newsprint, notebook paper, and art paper. Producing 1 roll of newprint requires 0.7 tons of wood pulp, 5 hours for processing, and 300 kilowatt hours of electricity. Producing 1 ton of notebook paper requires 1.5 tons of wood pulp, 6 hours for processing, and 500 kilowatt hours of electricity. Producing 1000 sheets of art paper requires 0.5 tons of wood pulp, 7 hours for processing, and 600 kilowatt hours of electricity. The company's available resources are 14 tons of wood pulp, 250 hours for processing, and 7,000 kilowatt hours of electricity. The profit for each roll of newsprint is \$ 250, the profit for each ton of notebook paper is \$ 200, and the profit for 1000 sheets of art paper is \$ 400. How much of each product should the company produce in order to maximize its profit, and what is the maximum profit?

FINAL – VERSION 1

1. The sets K, L, and M are all subsets of a universal set U, and it is known that $n(U) = 101$, $n(K) = 50$, $n(L) = 45$, $n(M) = 35$, $n(K \cap L) = 10$, $n(K \cap M) = 20$, $n(K \cap L \cap M) = 5$, and $n(K' \cap L' \cap M') = 10$. Find $n(L \cap M)$.

A) 9 B) 14 C) 24 D) 19

E) none of the others

2. Madonna flies on Nighthawk Airlines 40% of the time and on Dayglow Airlines 60% of the time. On Nighthawk she loses her luggage 20% of the time, and she loses it 10% of the time on Dayglow. If, on a randomly selected flight she did not lose her luggage, what is the probability that this flight was on Nighthawk?

A) 27/43 B) 16/43 C) .32 D) .86

E) none of the others

3. A Markov chain has the transition matrix $\begin{bmatrix} .4 & .6 \\ .2 & .8 \end{bmatrix}$ What is the long-run probability of being in state two?

A) 1/3 B) 1/4 C) 3/4 D) 2/3

E) none of the others

4. An experiment has the sample space $S = \{O_1, O_2, O_3, O_4\}$, and it is known that $w_1 = 3/8$, $w_2 = 2/8$, and $w_3 = 2/8$ (recall that $w_i = Pr[O_i]$). If $E = \{O_2, O_4\}$, what is $Pr[E]$?

A) 5/8 B) 3/8 C) 2/8 D) 1/8

E) none of the others

5. Suppose that E and F are independent events in a sample space S, with Pr[E] = .3 and Pr[F] = .4. What is $Pr[E \cap F']$?

A) .28 B) .18 C) .12 D) .42

E) none of the others

6. A Senate committee contains 4 Democrats, 3 Republicans, and one Independent. How many ways can a subcommittee of 4 be chosen, if the subcommittee must contain at least two Democrats?

A) 53 B) 36 C) 144 D) 52

E) none of the others

7. A pair of fair dice is rolled and the sum is noted. What is the probability that the sum is not 5, 7, or 9?

A) 7/18 B) 11/18 C) 7/12 D) 3/4

E) none of the others

8. A box contains 3 right gloves and 4 left gloves. A glove is drawn at random. If it is a left glove, it is returned to the box and two gloves are drawn at random (simultaneously). If it is a right glove, then it is kept out of the box and one more glove is randomly drawn. What is the probability that the final two gloves consist of one left glove and one right glove?

A) 3/7 B) 16/49 C) 30/49 D) 19/49

E) none of the others

9. Using the matrices **A, B, and C** shown below, find the entry in the second row and second column of **AB - 2C.**

$$A = \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & -1 & 1 \\ 0 & -2 & -2 \end{bmatrix} \qquad C = \begin{bmatrix} 5 & -1 & 2 \\ -3 & 2 & 0 \end{bmatrix}$$

A) 5 B) -15 C) 12 D) 1

E) none of the others

10. For which value of k does the matrix **A** not have an inverse?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & k \\ 0 & 3 & 6 \end{bmatrix}$$

A) 3 B) 6 C) 2 D) 4

E) none of the others

11. Find the minimum of $x - 2y$ on the feasible set given by the constraints: $x \geq 0$, $y \geq 0$, $x + y \geq 3$, $y \leq 3$, $x - y \leq 3$.

A) 3 B) 0 C) -9 D) -6

E) none of the others

12. Tom and Dick make two sizes of kites out of paper and wood strips. Each small kite uses 4 square feet of paper and 2 feet of wood strips, and each large kite uses 8 square feet of paper and 3 feet of wood strips. There are 160 square of paper and 70 feet of wood strips available. If all of the paper and all of the wood strips are used to make large and small kites, how many large kites will be made?

A) 200 B) 100 C) 20 D) 10

E) none of the others

Shown below are augmented matrices for several systems of equations. These matrices are close to reduced form. For each of these systems, decide which of the following statements is a correct description of the set of solutions of the system.

A) The system has an infinite number of solutions with one arbitrary variable.
B) The system has exactly three solutions.
C) The system is inconsistent, i.e., the system has no solution.
D) The system has a unique solution and $x_2 = 5$
E) The system has an infinite number of solutions with two arbitrary variables.
F) None of the others.

13.
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

14.
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 1 & 4 \\ 0 & 1 & 1 & 4 \end{array}\right]$$

15.
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 1 & 4 \\ 0 & 1 & 1 & -1 \end{array}\right]$$

16. Which of the following is a correct statement about the three lines given by the equations: $x + y = 2$, $y = 5$, $x + 2y = 7$.

A) Two of these lines are parallel.

B) Two of these lines have a positive slope, and the slope is undefined for the other line.

C) None of these lines are parallel, but they do not all go through the same point.

D) These lines all go through a single point.

E) None of the others

17. A system of equations is given in matrix form as **AX = B**, where **A**, **X**, **B**, and $\mathbf{A}^{-1}$ are shown below. Find y.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \quad \mathbf{A}^{-1} = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

A) 3 B) -2 C) -3 D) 2

E) none of the others

18. An experiment consists of flipping a fair coin three times and noting the number of heads and tails. A random variable X is defined to be two times the number of heads plus the number of tails. Find E[X], the expected value of X.

A) 4.0 B) 3.5 C) 5.0 D) 4.5

E) none of the others

19. Decide which (if any) of the following matrices is a transition matrix for an absorbing Markov chain.

I) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ II) $\begin{bmatrix} 1/2 & 1/2 \\ 0 & 1 \end{bmatrix}$ III) $\begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/3 & 0 & 2/3 \end{bmatrix}$

A) I and II B) I only C) II and III

D) II only E) none of the others

20. Freddie the Frog spends all his time on three lily pads, conveniently marked 1, 2, and 3. At the end of each minute, if he is on 1, he stays with probability .2 and jumps to 2 or 3 with equal probabilities. If he is on 2, he jumps to 1 with probability .9 and to 3 with probability .1. If he is on 3, he always jumps to 2. Assuming his moves obey the rules for a Markov chain, which of the following is the correct transition matrix?

A) $\begin{bmatrix} .2 & .5 & .5 \\ .9 & 0 & .1 \\ 0 & 1 & 0 \end{bmatrix}$ B) $\begin{bmatrix} .2 & .4 & .4 \\ .1 & 0 & .9 \\ 0 & 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} .2 & .4 & .4 \\ .9 & 0 & .1 \\ 1 & 0 & 0 \end{bmatrix}$

D) $\begin{bmatrix} .2 & .4 & .4 \\ .9 & 0 & .1 \\ 0 & 1 & 0 \end{bmatrix}$ E) none of the others

21. A Markov chain has the transition matrix shown below. Which of the statements below correctly describes this Markov Chain?

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ .5 & 0 & .5 \end{bmatrix}$$

A) The chain is regular and the stable vector is (1/4 1/4 1/2).
B) The chain is both regular and absorbing.
C) The chain is regular and the stable vector is (1/3 1/3 1/3).
D) The chain is regular and the stable vector is (1 1 2).
E) none of the others

22. Which of the following points is not a point of the feasible set determined by the constraints:
$x \geq 0$, $y \geq 0$, $x \geq y$, $x \leq 2y$, $x + 2y \geq 4$.

A) (0,0) B) (2,1) C) (100,100) D) (2,2)

E) none of the others

23. Backwoods Co-op makes trail mix out of raisins and dried apricots. A small package uses 4 ounces of raisins and 4 ounces of apricots, and a large package uses 8 ounces of raisins and 4 ounces of apricots. They have 240 ounces of raisins available, 160 ounces of apricots available, and the profit per package is $1.00 for small and $3.00 for large. Decide how many of each size package they should make to maximize profit. Which of the following is the maximum profit?

A) $90 B) $40 C) $80 D) $160

E) none of the others

24. The economy of the moon is limited to two goods, cheese and rocks. Using a Leontief model, with cheese the first good and rocks the second, the most recent technology matrix and production schedule are **A** and **X**, respectively, shown below. What was the external demand for cheese?

$$\mathbf{A} = \begin{bmatrix} .1 & .4 \\ .3 & .1 \end{bmatrix} \qquad \mathbf{X} = \begin{bmatrix} 20 \\ 10 \end{bmatrix}$$

A) 3 units B) 14 units C) 30/23 units D) 90/23 units

E) none of the others

25. A fair coin is flipped 4 times. What is the probability of getting exactly 2 heads, given that at least one head came up?

A) $1 - (1/2)^4$ B) 2/5 C) 3/5 D) $6(1/2)^4$

E) none of the others

26. E and F are subsets of the universal set U, with n(E) = 10, n(F) = 17, n(U) = 40, and n(E∩F) = 6. Find n(E′∩F′).

A) 36 B) 47 C) 11 D) 19

E) none of the others

27. Ross can take any of four flights from Chicago to Dallas, any of five flights from Dallas to Mexico City, and any of six direct flights back to Chicago from Mexico City. How many options are available for the trip Chicago-Dallas-Mexico City-Chicago?

A) 15! B) 15 C) 4!x5!x6! D) 120

E) none of the others

28. An experiment consists of flipping a fair coin until either two heads appear or there are four flips in all. An outcome consists of an ordered list of the results of each flip. For example, HTH is one possible outcome. How many outcomes exist?

A) 10 B) 9 C) 12 D) 11

E) none of the others

29. Bill and Hillary are planning a trip to Europe. They decide to visit five of the six cities: Paris, London, Vienna, Rome, Berlin, and Madrid. A trip itinerary consists of a list of five cities in the order they are to be visited. If Paris must be one of the cities visited, how many different itineraries are possible?

A) 5! B) 6! C) 2x4! D) 5x5!

E) none of the others

30. Which of the following statements correctly describes the feasible set determined by the constraints:
$x \geq 0,\ y \geq 0,\ 3x + 3y \geq 9,\ x - y \leq 2,\ -x + y \leq 2$

A) The feasible set is bounded with three corner points.
B) The feasible set is unbounded with two corner points.
C) The feasible set is bounded with two corner points.
D) The feasible set is unbounded with three corner points.
E) none of the others.

31. Two hundred students in Finite Mathematics are asked about their favorite parts of the course. 50 students say that they like probability problems, 60 students say that they like linear programming problems, and 20 students like both probability and linear programming problems. How many of these 200 students like linear programming problems, but not probability problems?

A) 40 B) 30 C) 20 D) 110

E) none of the others

32. Twenty students compete for four different prizes. If no student may receive more than one prize and if all prizes must be awarded, how many ways are there to award the prizes?

A) C(20,4) B) P(20,4) C) 4^{20} D) 20^4

E) none of the others

33. A Markov chain with two states has the transition matrix

$$\begin{bmatrix} .4 & .6 \\ .2 & .8 \end{bmatrix}$$

If this chain starts in state 2, what is the the probability it will be in state 2 after two transitions?

A) .12 B) .64 C) .76 D) .24

E) none of the others

FINAL – VERSION 2

1. (4 points) The president of the bank must select one person to manage the East Gate branch, another person to manage the North Mall branch, and a third person to manage the West Shore branch. She will choose these managers from a group of 9 assistant managers. How many different outcomes are possible?

(A)	19,683	(B)	524	(C)	504	(D)	729	(E)	27
(F)	84	(G)	24	(H)	1	(J)	none of the others		

2. (4 points) What is the entry in the second row and first column of the matrix obtained by computing

$$\begin{bmatrix} 3 & -4 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} -3 & 8 \\ 6 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & -7 \\ 4 & 5 \end{bmatrix} ?$$

(A)	0	(B)	13	(C)	−23	(D)	−24	(E)	24
(F)	8	(G)	−26	(H)	−36	(J)	41	(K)	12
(L)	48	(M)	30	(N)	none of the others				

3. (4 points) An interior decorator must select a color of paint for each of apartments A, B, and C. If he chooses from 5 shades of off-white, 4 shades of yellow, and 3 shades of blue, how many different outcomes are possible?

(A)	1,320	(B)	220	(C)	60	(D)	33	(E)	3
(F)	1,728	(G)	531,441	(H)	12	(J)	36	(K)	216
(L)	none of the others								

4. (4 points) A certain 3×3 matrix A has as its inverse the matrix

$$A^{-1} = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -1 & 0 \\ 0 & -1 & 2 \end{bmatrix} .$$

If

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \\ -3 \end{bmatrix} ,$$

then what is the value of x?

(A)	20	(B)	9	(C)	23
(D)	−1	(E)	−36	(F)	The system is inconsistent.
(G)	−31	(H)	−17	(J)	−10
(K)	1	(L)	Insufficient information	(M)	none of the others

5. (4 points) When the 47 partners of the law firm met to determine whether to promote John or Susan (or both) from associate to partner, 34 favored promoting John, 31 favored promoting Susan, and 5 opposed both promotions. How many favored promoting both John and Susan?

(A)	11	(B)	18	(C)	13	(D)	23	(E)	8
(F)	28	(G)	5	(H)	33	(J)	19	(K)	42
(L)	16	(M)	none of the others						

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6. (4 points) A purchasing manager for a construction company purchases gravel from three companies — Cox Gravel, Miller Rock, Smith Stone. In order to maintain competition between them and keep prices low, for each purchase she selects one of the suppliers at random according to the following rules: She never orders from the same supplier two times in succession. If one purchase is from Cox, then for the next purchase she is equally likely to select Miller or Smith. If one purchase is from Miller, then for the next purchase she is two times more likely to select Smith than to select Cox. If one purchase is from Smith, then for the next purchase she is three times more likely to select Miller than to select Cox. View this process as a Markov chain with state 1 being a purchase from Cox, state 2 being a purchase from Miller, and state 3 being a purchase from Smith. Which of the following matrices is the transition matrix of this Markov chain?

(A) $\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 0 & 2/3 \\ 0 & 1/4 & 3/4 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/3 & 2/3 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 2/3 & 3/4 \\ 1/2 & 0 & 1/4 \\ 1/2 & 1/3 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/3 & 0 & 2/3 \\ 1/4 & 3/4 & 0 \end{bmatrix}$ (E) $\begin{bmatrix} 0 & 1/3 & 1/4 \\ 1/2 & 0 & 3/4 \\ 1/2 & 2/3 & 0 \end{bmatrix}$ (F) $\begin{bmatrix} 0 & 1/2 & 1/2 \\ 0 & 1/3 & 2/3 \\ 0 & 1/4 & 3/4 \end{bmatrix}$

(G) $\begin{bmatrix} 0 & 1/2 & 1/2 \\ 2/3 & 0 & 1/3 \\ 3/4 & 1/4 & 0 \end{bmatrix}$ (H) $\begin{bmatrix} 1/2 & 1/3 & 0 \\ 1/2 & 0 & 1/4 \\ 0 & 2/3 & 3/4 \end{bmatrix}$ (J) none of the others

---

7. (4 points) 70 % of the Chamber's members favor mandatory health insurance and the rest oppose it. 80 % of those who favor mandatory health insurance favor welfare reform. 70 % of those who oppose mandatory health insurance favor welfare reform. If a member of the chamber is selected at random, what is the probability that this person does not favor welfare reform?

(A)	0.75	(B)	0.35	(C)	0.77	(D)	0.65	(E)	0.25
(F)	0.16	(G)	0.35	(H)	0.23	(J)	none of the others		

---

8. (4 points) A well known company makes two types of disposable diapers: a premium type (P) and a low cost type (L). Producing 1 gross (= 144) of type P diapers requires 48 pounds of cotton and 6 minutes for processing. Producing 1 gross of type L diapers requires 36 pounds of cotton and 5 minutes for processing. The net profit for one gross of type P diapers is \$ 7, and the net profit for one gross of type L diapers is \$ 4. The company has available this week 4,000 pounds of cotton and 500 hours for processing. The company wants to use these resources in a way that will maximize its profit. Let

u_1 = the number of type P diapers to be produced in units of 1 gross,

u_2 = the number of type L diapers to be produced in units of 1 gross,

s_1 = the amount of cotton to be used in units of pounds, and
s_2 = the amount of time to be used for processing in units of minutes.

The mathematical formulation of this problem contains which of the following expressions?

(I) $48u_1 + 36u_2 \leq 4,000$ (II) $6u_1 + 5u_2 \leq 500$
(III) Maximize $7s_1 + 4s_2$ (IV) $2s_1 + 500s_2 \geq 144$

(A)	None	(B)	I and III only	(C)	I, II, and III only
(D)	I only	(E)	I and IV only	(F)	I, II, and IV only
(G)	II only	(H)	II and III only	(J)	I, III, and IV only
(K)	III only	(L)	II and IV only	(M)	II, III, and IV only
(N)	IV only	(P)	III and IV only	(Q)	I, II, III, and IV
(R)	I and II only	(S)	none of the others		

9. (4 points) For which of the following transition matrices are the corresponding Markov chains regular or absorbing?

$$\text{(I)} \begin{bmatrix} 0 & 0 & 1 \\ 2/3 & 0 & 1/3 \\ 0 & 1 & 0 \end{bmatrix} \qquad \text{(II)} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 2/3 & 1/3 & 0 \end{bmatrix}$$

$$\text{(III)} \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 0 & 1/3 \\ 0 & 1 & 0 \end{bmatrix} \qquad \text{(IV)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2/3 & 1/3 \\ 0 & 1 & 0 \end{bmatrix}$$

(A)	None	(B)	I and III only	(C)	I, II, and III only
(D)	I only	(E)	I and IV only	(F)	I, II, and IV only
(G)	II only	(H)	II and III only	(J)	I, III, and IV only
(K)	III only	(L)	II and IV only	(M)	II, III, and IV only
(N)	IV only	(P)	III and IV only	(Q)	I, II, III, and IV
(R)	I and II only	(S)	none of the others		

10. (4 points) A museum's curator is to select 3 still lifes and 2 portraits by a certain artist for an anniversary show. The museum has 7 still lifes and 5 portraits by this artist. How many different outcomes are possible?

(A)	95,040	(B)	5	(C)	1	(D)	8,575	(E)	4,200
(F)	31	(G)	350	(H)	45	(J)	230	(K)	368
(L)	210	(M)	792	(N)	60	(P)	none of the others		

For each of the augmented matrices in the next three problems, determine which of the following statements is true about the associated system of linear equations:

(A) The system has no solution.
(B) The system has exactly one solution.

(C) The system has exactly two solutions.

(D) The system has exactly three solutions.

(E) The system has exactly four solutions.

(F) The system has infinitely many solutions in which one variable can be selected arbitrarily.

(G) The system has infinitely many solutions in which two variables can be selected arbitrarily.

(H) The system has infinitely many solutions in which three variables can be selected arbitrarily.

(J) None of the others is true.

---

11. (4 points) $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 1 & 4 \\ 1 & 2 & 0 & 12 \\ -1 & -1 & 1 & -7 \end{array}\right]$

---

12. (4 points) $\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ -1 & -3 & 1 & -2 & -1 \end{array}\right]$

---

13. (4 points) $\left[\begin{array}{cc|c} 1 & 2 & 5 \\ -1 & -1 & -4 \\ 2 & 2 & 8 \\ 1 & 3 & 6 \end{array}\right]$

---

14. (4 points) 77 % of a group of travelers have Visa cards, 66 % have American Express cards, and 55 % have both. If one of these travelers selected at random does not have an Visa card, what is the probability that this traveler also does not have a American Express card?

(A)	$\frac{1}{6}$	(B)	$\frac{11}{17}$	(C)	$\frac{12}{23}$	(D)	$\frac{11}{23}$	(E)	$\frac{2}{7}$
(F)	0.11	(G)	$\frac{5}{7}$	(H)	0.22	(J)	$\frac{5}{6}$	(K)	$\frac{6}{17}$
(L)	0.55	(M)	0.12	(N)	none of the others				

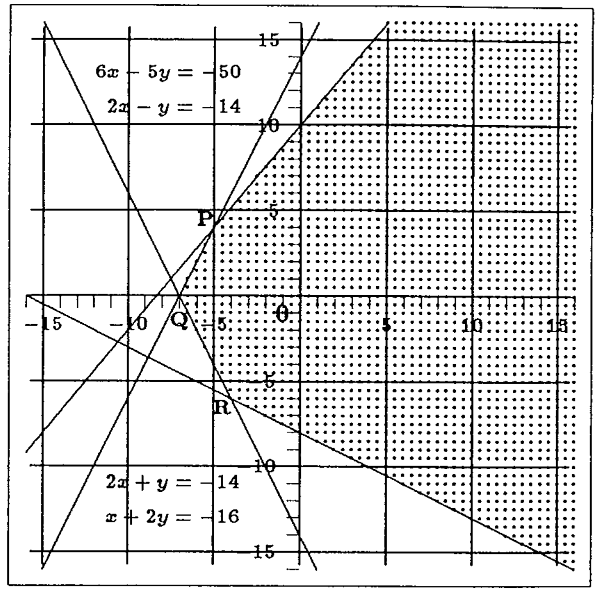
---

15. (4 points) Find the **minimum** value of $x + y$ subject to the constraints

$$6x - 5y \geq -50$$
$$2x - y \geq -14$$
$$2x + y \geq -14$$
$$x + 2y \geq -16$$

The feasible set for these constraints is graphed to the right. The corners are

P: $(-5, 4)$ Q: $(-7, 0)$
R: $(-4, -6)$



(A)	-12	(B)	-8	(C)	2	(D)	-9
(E)	No solution	(F)	8	(G)	-1	(H)	-10
(J)	-7	(K)	10	(L)	none of the others		

16. (4 points) A Markov chain has the transition matrix

$$P = \begin{bmatrix} 2/5 & 3/5 \\ 4/5 & 1/5 \end{bmatrix} .$$

Which of the following matrices is the three-step transition matrix for this chain?

(A) $\begin{bmatrix} 16/25 & 12/25 \\ 9/25 & 13/25 \end{bmatrix}$ (B) $\begin{bmatrix} 364/625 & 261/625 \\ 348/625 & 277/625 \end{bmatrix}$ (C) $\begin{bmatrix} 6/5 & 9/5 \\ 12/5 & 3/5 \end{bmatrix}$

(D) $\begin{bmatrix} 68/125 & 76/125 \\ 57/125 & 49/125 \end{bmatrix}$ (E) $\begin{bmatrix} 16/25 & 9/25 \\ 12/25 & 13/25 \end{bmatrix}$ (F) $\begin{bmatrix} 68/125 & 57/125 \\ 76/125 & 49/125 \end{bmatrix}$

(G) $\begin{bmatrix} 4/25 & 9/25 \\ 16/25 & 1/25 \end{bmatrix}$ (H) $\begin{bmatrix} 8/125 & 27/125 \\ 64/125 & 1/125 \end{bmatrix}$ (J) $\begin{bmatrix} 86/125 & 39/125 \\ 78/125 & 47/125 \end{bmatrix}$

(K) none of the others

17. (4 points) 7 men and 6 women, all equally qualified, apply for 3 positions as computer programmers. If 3 of these applicants are selected at random for the positions, what is the probability that exactly 2 women are selected?

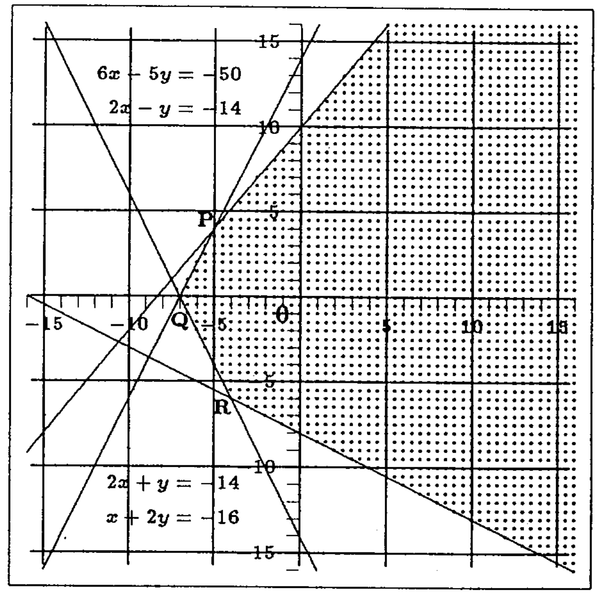
(A)	$\frac{6}{13}$	(B)	$\frac{1}{252}$	(C)	$\frac{35}{286}$	(D)	$\frac{105}{286}$	(E)	$\frac{19}{286}$
(F)	$\frac{756}{2,197}$	(G)	$\frac{1}{2}$	(H)	$\frac{252}{2,197}$	(J)	$\frac{43}{252}$	(K)	$\frac{19}{39}$
(L)	$\frac{1}{6}$	(M)	none of the others						

18. (4 points) 70 % of the audience enjoyed the play. 80 % of those who enjoyed the play liked the sets while 90 % of those who did not enjoy the play did not like the sets. If a member of the audience selected at random liked the sets, what is the probability that this person did not enjoy the play?

(A) $\frac{3}{59}$	(B) $\frac{14}{25}$	(C) $\frac{14}{17}$	(D) $\frac{56}{59}$	(E) $\frac{56}{83}$
(F) $\frac{27}{100}$	(G) $\frac{7}{50}$	(H) $\frac{27}{83}$	(J) $\frac{27}{41}$	(K) $\frac{3}{100}$
(L) $\frac{14}{41}$	(M) $\frac{3}{17}$	(N) none of the others		

19. (4 points) Find the **minimum** value of $x - y$ subject to the constraints

$$6x - 5y \geq -50$$
$$2x - y \geq -14$$
$$2x + y \geq -14$$
$$x + 2y \geq -16$$



The feasible set for these constraints is graphed to the right. The corners are

P: $(-5, 4)$ Q: $(-7, 0)$
R: $(-4, -6)$

(A) -12	(B) No solution	(C) 2	(D) -7
(E) -10	(F) -9	(G) -8	(H) -1
(J) 8	(K) 10	(L) none of the others	

20. (4 points) A Markov chain has the transition matrix

$$P = \begin{bmatrix} 0 & 2/5 & 3/5 \\ 1/5 & 4/5 & 0 \\ 2/5 & 1/5 & 2/5 \end{bmatrix}$$

and an initial state vector

$$X_0 = [1/6 \quad 1/3 \quad 1/2]\ .$$

What is the probability that this chain will be in state 1 after two transitions?

(A) $\frac{7}{25}$	(B) $\frac{3}{10}$	(C) $\frac{263}{750}$	(D) $\frac{77}{150}$	(E) $\frac{31}{150}$
(F) $\frac{4}{15}$	(G) $\frac{206}{375}$	(H) $\frac{161}{750}$	(J) $\frac{13}{30}$	(K) $\frac{8}{25}$
(L) none of the others				

21. (4 points) In a certain computer game, the player's first opponent is either a dragon or a monster. Each time the game is played, the computer randomly and independently selects whether the first opponent is a dragon or a monster, and the probability that it selects a dragon is $\frac{4}{7}$. If 4 people play this game, what is the probability that the first opponent is a dragon for 3 of them?

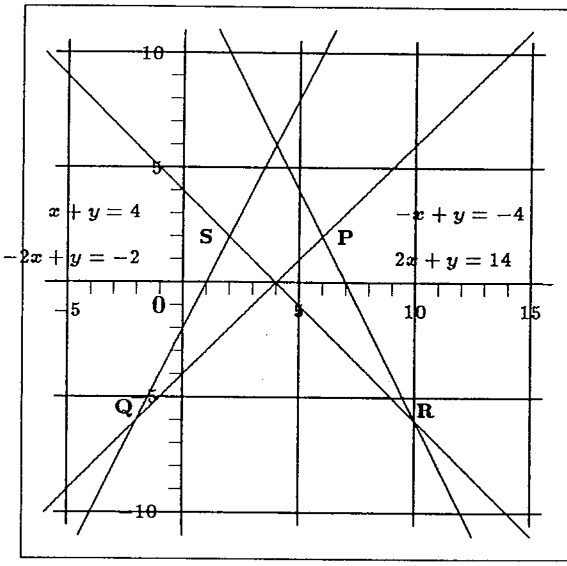
(A)	$\frac{192}{2,401}$	(B)	$\frac{108}{2,401}$	(C)	$\frac{9}{7}$	(D)	$\frac{27}{343}$	(E)	$\frac{432}{2,401}$
(F)	$\frac{768}{2,401}$	(G)	$\frac{9}{28}$	(H)	$\frac{4}{7}$	(J)	$\frac{3}{4}$	(K)	$\frac{3}{7}$
(L)	$\frac{12}{35}$	(M)	$\frac{64}{343}$	(N)	$\frac{1}{7}$	(P)	$\frac{6}{49}$	(Q)	none of the others

22. (4 points) Which of the following points are corner points of the set described by the system of inequalities

$$x + y \geq 4$$
$$-x + y \geq -4$$
$$2x + y \leq 14$$
$$-2x + y \leq -2$$

R: $(10, -6)$ P: $(6, 2)$
S: $(2, 2)$ Q: $(-2, -6)$

(See the graph to the right.)



(A)	None	(B)	P and R only	(C)	P, Q, and R only
(D)	P only	(E)	P and S only	(F)	P, Q, and S only
(G)	Q only	(H)	Q and R only	(J)	P, R, and S only
(K)	R only	(L)	Q and S only	(M)	Q, R, and S only
(N)	S only	(P)	R and S only	(Q)	P, Q, R, and S
(R)	P and Q only	(S)	none of the others		

23. (4 points) A bin contains 7 rolls of Kodak film and 5 rolls of Fuji film. If a customer selects 4 rolls simultaneously from this bin, how many different possible outcomes have at least one roll of Kodak film and at least one roll of Fuji film?

(A)	495	(B)	485	(C)	20,736	(D)	69	(E)	10,920
(F)	1,575	(G)	34	(H)	455	(J)	3,500	(K)	3,150
(L)	545	(M)	11,880	(N)	33	(P)	none of the others		

24. (4 points) A regular Markov chain has the transition matrix

$$P = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 1/3 & 2/3 & 0 \\ 0 & 2/3 & 1/3 \end{bmatrix}.$$

What is the first entry of its stable vector?

(A)	$\frac{6}{13}$	(B)	$\frac{1}{4}$	(C)	$\frac{2}{9}$	(D)	$\frac{4}{9}$	(E)	$\frac{1}{6}$
(F)	$\frac{3}{13}$	(G)	$\frac{4}{13}$	(H)	$\frac{1}{3}$	(J)	$\frac{4}{17}$	(K)	$\frac{2}{11}$
(L)	$\frac{1}{2}$	(M)	$\frac{2}{15}$	(N)	$\frac{3}{10}$	(P)	$\frac{2}{3}$	(Q)	$\frac{4}{11}$
(R)	$\frac{5}{11}$	(S)	$\frac{9}{20}$	(T)	none of the others				

25. (4 points) A contractor is building an office building and an apartment building for the Jones Corporation. He will receive a bonus of \$ 30,000 if he finishes both buildings a month early. He will receive a bonus of \$ 15,000 if he finishes the office building but not the apartment building a month early. He will receive a bonus of \$ 10,000 if he finishes the apartment building but not the office building a month early. The contractor estimates that the probability of finishing the office building a month early is 0.45, the probability of finishing the apartment building a month early is 0.50, and the probability of finishing neither building a month early is 0.25. What is the contractor's expected bonus (to the nearest cent)?

(A)	12,875.00	(B)	18,333.33	(C)	12,750.00	(D)	19,250.00
(E)	17,750.00	(F)	34,250.00	(G)	18,000.00	(H)	14,250.00
(J)	13,000.00	(K)	18,500.00	(L)	none of the others		

SAMPLE QUIZZES

This section contains quizzes collected from previous M118 classes. Allow 10–15 minutes to work on each one.

QUIZ 1

1. Consider the universal set $S = \{1, 2, 3, 4, 5, a, b, c, d\}$ with the subsets $N = \{1, 2, 3, 4, 5, a\}$, $L = \{a, b, c, d, 1\}$, and $M = \{1, 3, 5, a, b\}$. List the elements in each of the following sets:
   a) $N \cap M'$
   b) $L \cup N'$
   c) $N \cap (M \cup L)$
   d) $(M \cup L') \cap N'$

2. At Indiana University, the typical class of new students has 6000 members. In the current class of new students, 2600 plan to major in the College of Arts and Sciences, 2200 plan to major in Business, and 700 plan to major in Music. Several of these students plan to be double majors; in fact 300 of those who plan to major in Arts and Sciences also plan to major in Business, 150 of those who plan to major in Arts and Sciences also plan to major in Music, and 25 of those who plan to major in Music also plan to major in Business. No student plans to major in all three of these schools.
   a) How many students do not plan to major in Arts and Sciences, or Business, or Music?
   b) How many students plan to major in Music but not Business?

QUIZ 2

Suppose that a certain class has 40 students and each of them is to be assigned to a study session. These sessions are to be at 4, 5, or 6 pm, and they are on Mon., Tues., Wed., Thurs., or Fri. For example, a student might be assigned Tuesday at 4.

a) How many different ways can a single student be assigned?
b) How many different ways can all 40 students be assigned?
c) How many different ways can all 40 students be assigned if no student is to be assigned Friday at 6 pm?

QUIZ 3

1. In a certain dorm, 60% of the residents are freshmen, and 40% are sophomores. 70% of the freshmen take mathematics and 40% of the sophomores take mathematics. What is the probability that a randomly selected student takes mathematics?

2. A, B and C are events in a sample space S. $\Pr[A] = 0.42$, $\Pr[B] = 0.62$, $\Pr[C] = 0.38$, $A \cap C = \emptyset$, and $\Pr[(A \cup B \cup C)'] = 0.07$. What is the probability that a sample point of S is in exactly one of A, B, and C?

3. Some of the farms in Lipton county need new barns and some need new fences. The probability that a farm needs a new barn is 0.75. The probability that a farm needs a new fence is 0.50. The probability that a farm is in good condition (that is, needs neither a new barn nor new fences) is 0.15. What is the probability that a farm needs a new barn given that it needs a new fence?

QUIZ 4

1. Red of Red's Sport Shop orders his soccer balls from three suppliers. He gets 50% from Pittsburgh, 10% from Atlanta, and 40% from Dallas. He knows that some of these balls are defective. In fact, 5% from Pittsburgh, 15% from Atlanta, and 20% from Dallas are defective. Suppose a ball is selected at random.
   a) What is the probability that the ball is defective?
   b) If the ball selected is defective, what is the probability that it came from Atlanta?
2. An experiment consists of repeatedly rolling a fair die and checking if the outcome is either a 1 or a 6. There are 8 rolls in all.
   a) What is the probability that exactly four of the outcomes are a 1 or a 6?
   b) What is the probability that at least one of the outcomes is a 1 or a 6?

QUIZ 5

A game consists of rolling a pair of fair dice and noting the sum. If the sum is 8, 9, or 10, you receive the sum. If the sum is anything else, you lose 3$. Define a random variable X to be the amount you win (or lose).

a) Find the probability density for X.

b) Find the expected value of X.

QUIZ 6

1. a) Find the equation of the line passing through the pair of points (2,1) and (1,5).
   b) Find the line through (0,7) which is parallel to the line from part a).
2. Brown Brothers Box Business produces standard and heavy-duty boxes. Standard boxes require one square foot of cardboard and four square feet of liner board. Heavy-duty boxes require three square feet of cardboard and one square foot of liner board. They have supplies of 100 square feet of cardboard and 400 square feet of liner board. How many of each type of box should they make if they want to use up all their supplies?

QUIZ 7

Consider the following system of equations:

$$\begin{aligned} -2y + z &= 3 \\ x - 10y + 5z &= 14 \\ x - 6y + 3z &= 8 \end{aligned}$$

a) Write the augmented matrix and show reduced form.

b) Find all solutions of this system.

QUIZ 8

Let

$$A = \begin{bmatrix} 3 & 0 & 7 \\ 0 & 1 & 0 \\ 2 & 0 & 5 \end{bmatrix}.$$

Find A^{-1} or show it does not exist.

QUIZ 9

Consider a small economy in which there are two goods – coal and steel. The production of one unit of coal requires 0.4 units of coal and 0.3 units of steel. The production of one unit of steel requires 0.5 units of coal and 0.6 units of steel.

1. Find the technology matrix for this economy.
2. Find the inverse needed to compute the production schedule.
3. Find the production schedule needed to meet an external demand for 90 units of coal and 45 units of steel.

QUIZ 10

Formulate a linear programming problem for the situation described below. **You do not need to solve the problem, but be sure to state the formulation carefully.**

A fish farmer owns a 170 acre fish farm and can grow any combination of three fish: A, B, and C. Fish A requires 1 person-day of labor and \$75 of capital per acre farmed, fish B requires 3 person-days of labor and \$65 of capital per acre farmed, and fish C requires 1 person-day of labor and \$55 of capital per acre farmed. The farmer has \$75000 in capital and 260 person-days of labor. Also, the profit per acre of each fish is \$295, \$265, and \$225 for A, B, and C respectively. How many acres should be planted with each fish to maximize profit?

QUIZ 11

a) Find the maximum value of $z = x + 2y$ subject to the constraints

$$\begin{aligned} x &\leq 6 \\ y &\geq -4 \\ x - y &\leq 7 \\ x + y &\leq 8 \\ x + 2y &\geq -10. \end{aligned}$$

b) Find the minimum value of $z = x + 3y$ subject to the same constraints.

QUIZ 12

The matrix P given below is the transition matrix for a Markov chain. Suppose this chain starts in state 1. Use a tree diagram (show the tree) to find all of the two-step transition probabilities which begin at state 1.

$$P = \begin{bmatrix} .4 & .3 & .3 \\ 0 & 0 & 1 \\ .3 & .7 & 0 \end{bmatrix}$$

QUIZ 13

Let a Markov chain have transition matrix

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}.$$

1. Determine whether this Markov chain is regular. Show your derivation.
2. Find the stable vector W for this Markov chain.

ANSWER KEYS AND SOLUTIONS

This section contains answer keys to all of the quizzes and exams in this study guide. For some exams, hints and solutions are also included.

KEY
ANSWERS TO SUPPLEMENTARY PROBLEMS FOR SECTION 9.3

1. [4/20 9/20 7/20] = [1/5 9/20 7/20]

2. [11/21 4/21 6/21] = [11/21 4/21 2/7]

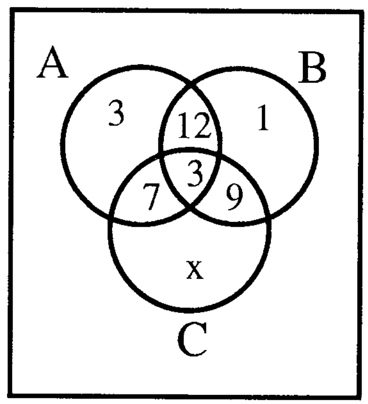
3. [5/22 6/22 11/22] = [5/22 3/11 1/2]

4. [29/74 28/74 17/74] = [29/74 14/37 17/74]

5. [27/79 32/79 20/79]

KEY
EXAM I – VERSION 1

1. (e) none of the others



2. (c) 27

Partition S into A and B where A is the event that the first roll is even and B is the event that the first roll is odd. Using the multiplication principle $n(\text{A}) = 3 \cdot 6 = 18$, because there are 3 outcomes in which the first roll is even and 6 outcomes for the result of the second roll. Similarly $n(\text{B}) = 9$ because there are 3 outcomes in which the first roll is odd, and 3 possible results for the number of heads ($\{0, 1, 2\}$). So $n(\text{S}) = n(\text{A}) + n(\text{B}) = 27$.

3. (b) 6^4

Multiplication principle.

4. (a) $\frac{1}{4}$

Let w_1 be the probability of "heads" and w_2 be the probability of "tails". Since "heads" come up one-third as often as "tails", $3w_1 = w_2$. From $w_1 + w_2 = 1$ we obtain (after substitution) $w_1 + 3w_1 = 1$. Solving this yields $w_1 = \frac{1}{4}$.

5. (d) 900

There are $C(6,2) = 15$ ways of picking 2 names from a list of 6. There are $C(5,2) = 10$ ways of picking 2 jobs from a list of 5 jobs. There are $C(4,2) = 6$ ways of picking 2 cities from a list of 4 cities. By multiplication principle there are $15 \cdot 10 \cdot 6$ outcomes for this experiment.

6. (d) $P(8,5)$

A "word" is an ordered list of elements from the set of letters (*i.e.* a permutation).

7. (a) $\frac{8}{9}$

There are 36 equally likely outcomes. There are 32 outcomes for which the product is less than 25, so the probability that the product is less than 25 is $\frac{32}{36} = \frac{8}{9}$.

8. (d) 36

Combinations.

9. (a) $\frac{15}{28}$

There are $C(8,3) = 56$ ways to choose three coins from the purse. The only choice which adds up to 25 cents is 2 dimes and one nickel. There are $C(5,2) \cdot C(3,2) = 30$ ways of choosing two dimes and a nickel from the purse. So the probability that the value of the coins is 25 cents is $\frac{30}{56} = \frac{15}{28}$.

10. (a) 60

Partition the set of choices into two subsets: choices in which a man is chair, and choices in which a woman is chair. If a man is chair, then a woman must be vice-chair, and there are $P(5,1) \cdot P(6,1) = 30$ ways in which this assignment can be made. Similarly, there are $P(6,1) \cdot P(5,1) = 30$ choices for which a woman is chair. Therefore the number of ways in which the offices can be filled is 60.

11. (d) $C(6,2) \cdot C(4,2)$

Multiplication principle.

12. (c) $\frac{1}{2^2}$

The set of possible guesses on the two questions for which Sam does not know the answer is {TT,TF,FT,FF}. Each outcome is equally likely, so the probability of getting both correct (TT) is $\frac{1}{4}$.

13. (c) 15

$Pr[\mathrm{E}] = \frac{n(\mathrm{E})}{n(\mathrm{S})}$. So $.3 = \frac{n(\mathrm{E})}{50}$, or $n(\mathrm{E}) = 15$.

14. (d) $\frac{5}{6}$

There are 36 possible outcomes from rolling two dice. Of these, exactly 6 are of the form (1,1), (2,2) etc.. This leaves 30 outcomes with different numbers on each roll, so the probability that the numbers are different is $\frac{30}{36} = \frac{5}{6}$.

15. (b) $\frac{1}{5}$

Let w_i be the weight of σ_i. Then $w_1 = w_2$ and $w_3 = 3w_1$. Substituting into $w_1 + w_2 + w_3 = 1$ yields $w_1 + w_1 + 3w_1 = 1$. Solving this gives $w_1 = \frac{1}{5}$, and $w_2 = w_1$.

16. (b) 3

Elementary set theory.

17. (c) $(A \cup B)' = A' \cap B'$ This is one of DeMorgan's laws.

18. (d) $x \notin B \cup C$ Elementary set theory.

19. (b) 38 The sets $(A \cup B)$ and $(A \cup B)'$ form a partition of U. We are given $n(A \cup B) = 28$. Since $(A \cup B)' = A' \cap B'$, we have $n((A \cup B)') = n(A' \cap B') = 10$. Therefore $n(U) = 28 + 10 = 38$.

20. (e) none of the others Let B={like to watch basketball} and F={like to watch football}. We want to find $n(B \cap F)$. Since 8 dislike both, we know that $n(B \cup F) = 100 - 8 = 92$. So $n(B \cap F) = n(B) + n(F) - n(B \cup F) = 87 + 21 - 92 = 16$.

KEY
EXAM I – VERSION 2

1. E	2. C	3. F	4. E	5. A
6. G	7. B	8. D	9. F	10. E
11. A	12. C	13. D	14. B	15. G
16. E	17. C	18. D	19. A	20. E

KEY
EXAM II

Multiple Choice

1. a	2. c	3. b	4. e	5. d
6. a	7. a	8. c	9. b	10. c
11. a	12. d	13. d	14. c	15. c

True/False

1. True	2. False	3. False	4. True	5. False

KEY
MIDTERM – VERSION 1

1. D

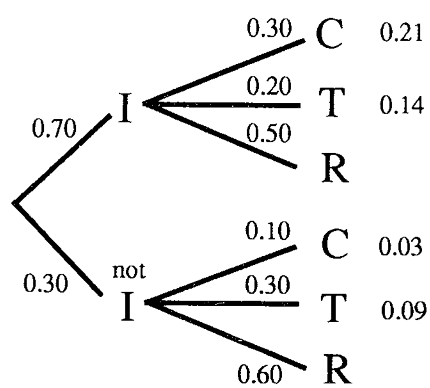
Let B denote voters supporting budget reform and H denote voters supporting health care reform. We are looking for $\Pr[B \cup H] - \Pr[B \cap H]$. Since 23% of voters oppose both reforms, $\Pr[B \cup H] = 1.00 - 0.23 = 0.77$. Then $\Pr[B \cap H] = \Pr[B] + \Pr[H] - \Pr[B \cup H] = 0.29 + 0.52 - 0.77 = 0.04$. Therefore, $\Pr[B \cup H] - \Pr]B \cap H] = 0.77 - 0.04 = 0.73$.

2. C

View this as a Bernoulli trial with success being the assignment of a female agent. Then $p = \frac{3}{5}$ and $n = 400$. Using the formula for the expected value of a Bernoulli trial we obtain $E[X] = np = 240$.

3. G

The data in the problem leads to the following tree diagram.



Therefore $\Pr[\text{CorT}] = .21 + .14 + .03 + .09 = .47$.

4. J

Permutation principle.

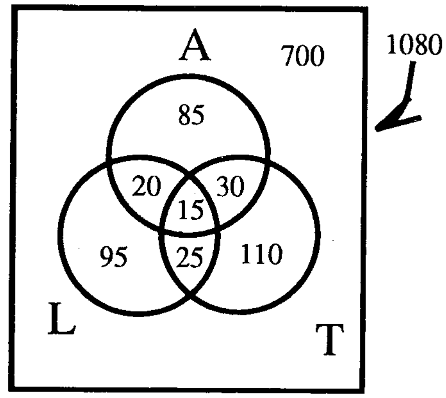
5. B

X has probability density function:

X	$\Pr[X = x_i]$	Product
-10	0.60	-6
30	0.20	10
100	0.14	14
200	0.05	10
500	0.01	5
		$E[X] = 33.$

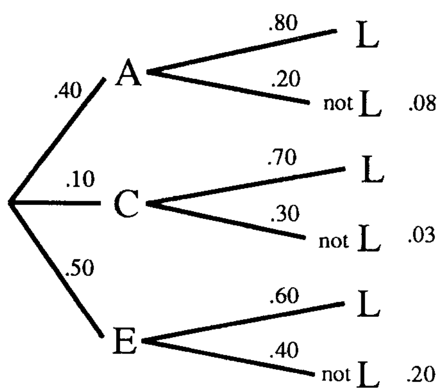
6. E

Use the following Venn diagram.



7. F

The data in the problem leads to the following tree diagram.



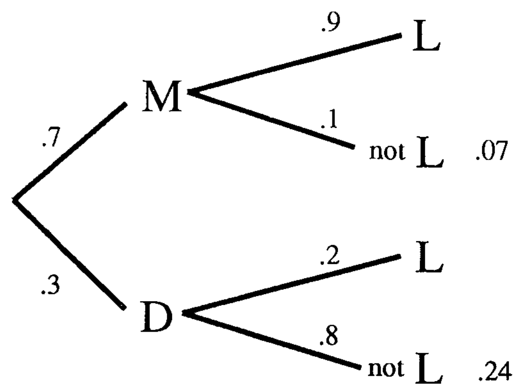
Therefore Pr[not legally parked] $= .08 + .03 + .2 = .31$.

8. B

An outcome can be viewed as an ordered triple where the first entry is John's nomination for first prize, the second is Elizabeth's and the third is Thomas'. Repititions are permitted, so the number of possible outcomes is $13 \cdot 13 \cdot 13 = 2,197$.

9. H

The data in the problem leads to the following tree diagram.



Therefore $\Pr[\mathrm{M}|\mathrm{notL}] = \dfrac{\Pr[\mathrm{M} \cap \mathrm{notL}]}{\Pr[\mathrm{notL}]} = \dfrac{.07}{.07 + .24} = \dfrac{7}{31}$.

10. A

The number of possible selections is $C(11,3) = 165$. The number of selections satsifying the condition "1 resident from each city is selected" is $C(2,1) \cdot C(7,1) \cdot C(2,1) = 28$. Since each selection is random (equally likely) the probability of satisfying the condition is $\frac{28}{165}$.

11. C

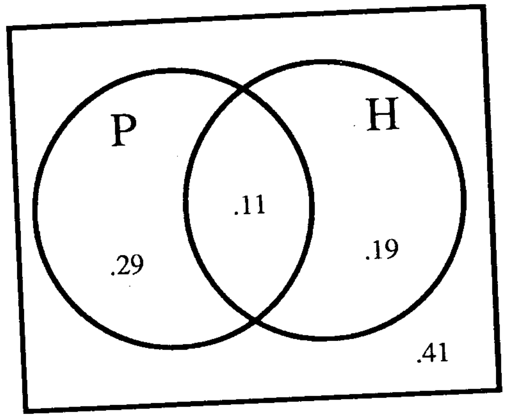
Let A denote "owns an imported automobile" and T denote "owns an imported television". We are looking for $\Pr[(\mathrm{A} \cup \mathrm{T})']$. First observe that $\Pr[\mathrm{A} \cup \mathrm{T}] = \Pr[\mathrm{A}] + \Pr[\mathrm{T}] - \Pr[\mathrm{A} \cap \mathrm{T}]$. Independence implies $\Pr[\mathrm{A} \cap \mathrm{T}] = \Pr[\mathrm{A}] \cdot \Pr[\mathrm{B}]$. Therefore, $\Pr[\mathrm{A} \cup \mathrm{T}] = 0.3 + 0.8 - (0.3)(0.8) = 0.86$. Finally, $\Pr[(\mathrm{A} \cup \mathrm{T})'] = 1 - Pr[\mathrm{A} \cup \mathrm{T}] = 0.14$.

12. H

The number of possible ways in which three of them can respond to a call is $C(8,3) = 56$. There are $C(5,1) \cdot C(3,2) = 15$ possible selections having only one man. Since the selection is random, The probability of having exactly one man is $\frac{15}{56}$.

13. E

If P denotes "wants to take a psychology course" and H denotes "wants to take a history course" then we are looking for Pr[P|H′]. Using the following Venn diagram to compute the relevant probabilities we obtain Pr[P ∩ H′] = 0.29 and Pr[H′] = 0.70. Therefore, $\Pr[P|H'] = \frac{\Pr[P\cap H']}{\Pr[H']} = \frac{.29}{.29+.41} = \frac{29}{70}$.

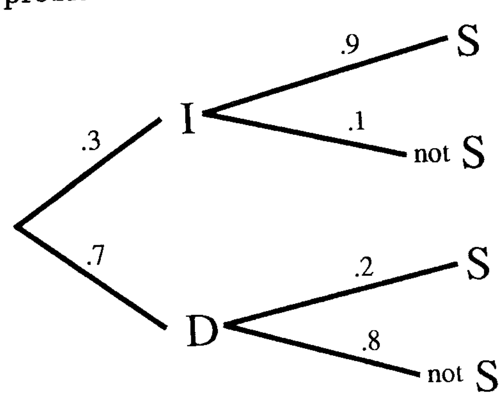


14. D

Multiplication principle.

15. F

The data in the problem leads to the following tree diagram.



Then, $\Pr[D|S] = \frac{\Pr[D\cap S]}{\Pr[S]} = \frac{.14}{.27+.14} = \frac{14}{41}$

16. G

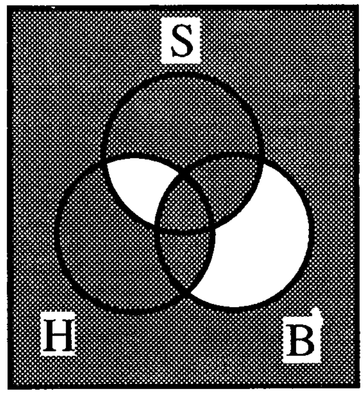
View this as a Bernoulli process with success being the selection of the square button. Then $p = \frac{1}{3}$, $r = 2$, and $n = 5$.

17. B

The number of possible selections is C(18, 2) = 153. The number of selections satisfying the condition "1 will be a North American company and 1 will be a European company" is C(10, 1) · C(8, 1) = 80. Since the selection is random, the probability of satisfying the condition is $\frac{80}{153}$.

18. J

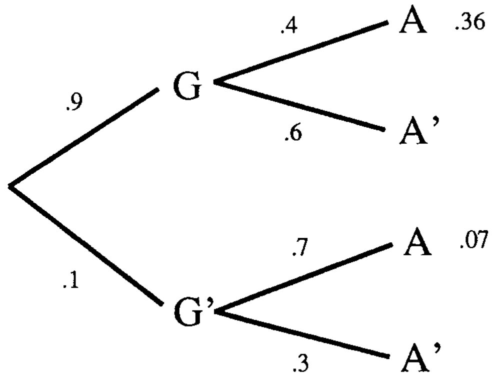
The set of interest is indicated in the Venn diagram below.



Use the original diagram to compute the probability.

19. G

Let G denote the event "favors gun control" and A denote the event "owns a gun". Then we obtain the following tree diagram.



Therefore, $\Pr[G|A] = \frac{\Pr[G \cap A]}{\Pr[A]} = \frac{.36}{.36+.07} = \frac{36}{43}$.

20. A

View this as a Bernoulli trial with success being the selection of a dark colored T-shirt. Then $p = \frac{3}{4}$ and $n = 4$. Use the fact that

$$\Pr[\leq 3 \text{ successes}] = 1 - \Pr[\text{exactly } 4 \text{ succeses}].$$

21. D

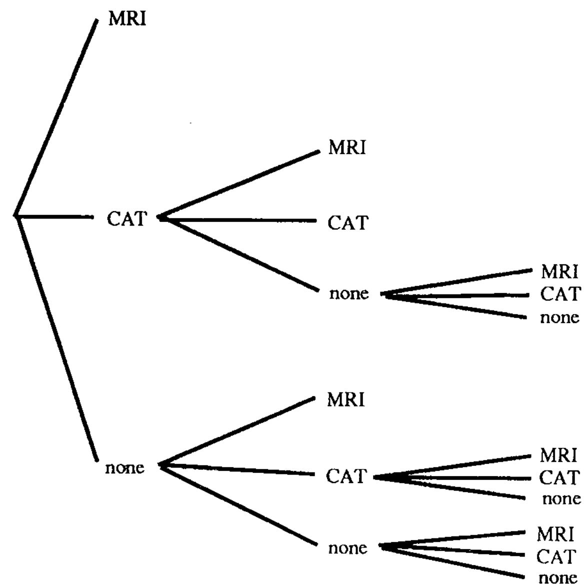
If A is the event "1 male and 1 female are selected" and B is the event "no male skiers are selected" then we are looking for $\Pr[A|B]$. Observe that $n(A \cap B) = C(4,1) \cdot C(11,1) = 44$ (choose 1 of the 4 non-skier males and 1 of the 11 females) and $n(B) = C(15,2) = 105$. Therefore, $\Pr[A|B] = \frac{44}{105}$.

22. F

Partition the set of all such boards into A=those with no electricians, B=those with 1 electrician, and C=those with at least two electricians. Then $n(A) = C(7,4) = 35$, $n(B) = C(7,3) \cdot C(5,1) = 175$ and $n(A \cup B \cup C) = n(\text{all possible boards}) = C(12,4) = 495$. Therefore, $n(C) = n(A \cup B \cup C) - n(A) - n(B) = 495 - 35 - 175 = 285$.

23. B

There are 13 possible outcomes.



KEY
MIDTERM – VERSION 2

1. C	2. B	3. D	4. E	5. C
6. A	7. C	8. D	9. A	10. D
11. D	12. E	13. C	14. B	15. E
16. D	17. E	18. D	19. E	20. A
21. A	22. A	23. B	24. E	25. D

KEY
EXAM III – VERSION 1

1. (b) $\frac{1}{2}$ — Compute.

2. (e) none of the others — The solution is $x = 1$, $y = 1 - z$, z arbitrary.

3. (d) $k = 6$ — Compute. Note that when $k = 6$, the third row is the sum of the first and second rows. This leads to a contradiction when one tries to compute the inverse.

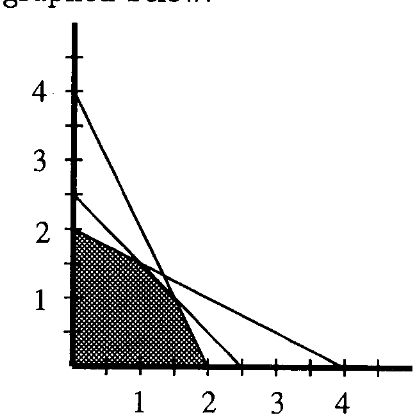
4. (c) the system has an infinite number of solutions and for all solutions $x = 2$. — Compute.

5. (b) 25 sweet, 50 tart — Let x be the number of glasses of sweet drink, and y the number of glasses of tart drink. Then the equations for the number of oranges and lemons used are

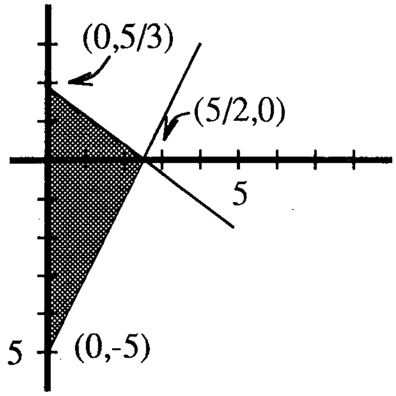
$$.8x + .6y = 50$$
$$.2x + .7y = 40.$$

Solving this system gives the answer.

6. (d) $(\frac{4}{3}, \frac{4}{3})$ — The feasible set is graphed below.



7. (e) none of the others — Evaluate the objective function at the corner points of the feasible set graphed below.



8. (c) no minimum exists — Points of the form $(n,0)$ are in the feasible set for n arbitrarily large. At $(n,0)$ the objective function takes the value $-n$.

9. (a) -26 — Compute.

10. (d) $\begin{bmatrix} -2 & -4 \\ 1 & 2 \end{bmatrix}$ — Compute.

11. (d) $2x - y = 4$ — Put each line in point slope form.

12. (a) $3x - y = 4$ — "Parallel" implies that the lines have the same slope. The slope of the line through the given points is $\frac{6-3}{2-1} = 3$.

13. (d) $\frac{10}{11}$ — Compute.

14. (b) $(3,-3)$ — This point does not satisfy the constraint $2x + y \geq 4$.

15. (e) none of the others — Elementary matrix algebra.

16. The solution is

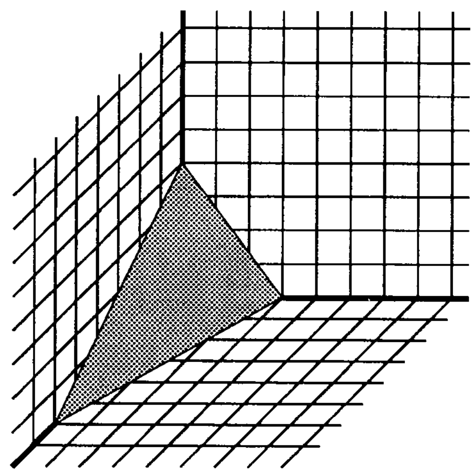
$$\begin{aligned} x &= -5 + 2y - 4w \\ y &= \text{ arbitrary} \\ z &= 6 + 3w \\ w &= \text{ arbitrary.} \end{aligned}$$

KEY
EXAM III – VERSION 2

1. $y = -3x + 19$

   Parallel lines have the same slope. Using the last two points, compute the slope: $m = \frac{-17-10}{4--5} = \frac{-27}{9} = -3$. Now plug the point $(4, 7)$ into the equation $y = -3x + b$ and solve for b, the y-intercept.

2. x-intercept: 6
   y-intercept: 3
   z-intercept: 4



3. $\begin{bmatrix} 16 & -32 \\ -18 & -47 \\ 21 & 46 \end{bmatrix}$

   Compute.

4. $\begin{bmatrix} -11 & -1 \\ 15 & 11 \end{bmatrix}$

   Compute.

5. (a) undefined (b) defined (c) undefined (d) undefined (e) undefined

6. (a) yes (b) no (c) yes (d) no

7. (a) no (b) yes (c) no (d) yes

8. B

   Compute the reduced row echelon form.

9. D

   Compute the reduced row echelon form.

10. A — Compute the reduced row echelon form.

11. E — Compute the reduced row echelon form.

12.(i) Let

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

where

$$\begin{aligned} x &= \text{amount of Normal } (100l) \\ y &= \text{amount of High Potency } (100l) \\ z &= \text{amount of Lo-Cal } (100l). \end{aligned}$$

Let

$$A = \begin{bmatrix} 16 & 12 & 7 \\ 12 & 3 & 9 \\ 5 & 4 & 6 \\ 50 & 20 & 40 \end{bmatrix}.$$

Then

$$AX = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

where

$$\begin{aligned} b_1 &= \text{liters of nectar A} \\ b_2 &= \text{kilograms of chemical B} \\ b_3 &= \text{liters of preservative C} \\ b_4 &= \text{minutes of processing time} \end{aligned}$$

12.(ii) 200 l of nectar A, 156 kg of chemical B, 94 l of preservative C and 700 min. of processing time. — Compute using setup from 12(i).

13. $\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$

14. $x_4 = 4$, $x_3 = 1$, $x_2 = t$ and $x_1 = -2 - 5t$ where t is arbitrary.

15. 50 of type A, 80 of type B and 40 of type C (all in units of 100 sq. ft.)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A^{-1} \begin{bmatrix} 330 \\ 740 \\ 1360 \end{bmatrix}$$

where A^{-1} is the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 4 & 8 & 13 \end{bmatrix}.$$

KEY
EXAM IV

1. -27 at A — Check each corner point.

2. 47 at B — Check each corner point.

3. 38 at D — D gives the largest value of all the corner points. Check a point on the edge near D on the unbounded side to verify that it is a maximum.

4. no maximum — The candidate for a maximum is A where $z = 29$. Observe that at (0,10.5) $z = 31.5$, so A is not a maximum.

5. -26 at B — Checking the corner points we find that B is a candidate. Since the candidate occurs only at a corner all of whose adjacent boundaries are bounded, the candidate is the solution.

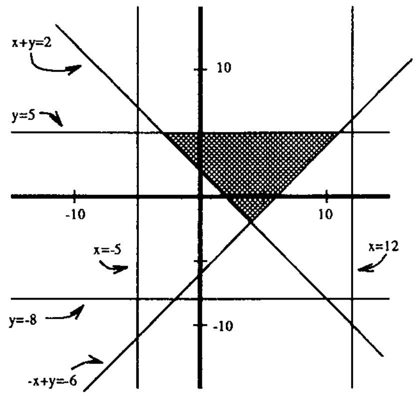
6. $\begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$ — Elementary matrix computation. If AX = B, then $X = A^{-1}B$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 25 \\ 15 \\ -9 \end{bmatrix}$.

7. $\begin{bmatrix} -1 & -2 & -2 \\ 1 & 1 & 1 \\ -3 & 0 & 1 \end{bmatrix}$

8.(a) $\begin{bmatrix} 360 \\ 520 \end{bmatrix}$

8.(b) $\begin{bmatrix} 320 \\ 240 \end{bmatrix}$

9. The graph is



The corner points occur at (4,-2), (11,5), and (-3,5).

10.(a) $\begin{bmatrix} 158/343 & 185/343 \\ 222/343 & 121/343 \end{bmatrix}$

10.(b) $\frac{222}{343}$ The probability of a transition froom state 2 to state 1 after 3 observations is the (2,1) entry of P(3).

11.(a) $[\ \frac{23}{147} \quad \frac{74}{147} \quad \frac{50}{147}\ ]$ $X_2 = X_1P = (X_0P)P$.

11.(b) $\frac{23}{147}$ The probability of being in state 1 after 2 steps is the first entry in X_2.

12. $[\ \frac{5}{37} \quad \frac{24}{37} \quad \frac{8}{37}\ ]$ Solve the matrix equation $W(P - I) = 0$ where $W = [\,w_1 \quad w_2 \quad w_3\,]$, P is as given and I is the 3×3 identity matrix. Then W is the stable vector.

13. Let

Good 1 = aluminum ore in tons

Good 2 = aluminum in tons

Good 3 = electricity in kilowatt-hours.

Then the technology matrix is

$$\begin{bmatrix} 0 & 4 & 0 \\ 0.001 & 0 & 0.002 \\ 50 & 200 & 0.03 \end{bmatrix}.$$

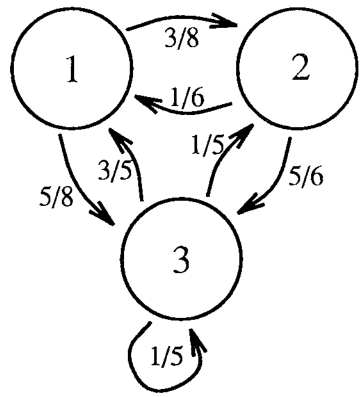
14. Let

state 1 =pepperoni,

state 2 =vegetarian,

state 3 =inferno.

Then the transition diagram is:



The transition matrix is

$$\begin{bmatrix} 0 & \frac{3}{8} & \frac{5}{8} \\ \frac{1}{6} & 0 & \frac{5}{6} \\ \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \end{bmatrix}.$$

15. Let

x =rolls of newsprint,

y =tons of notebook paper,

z =sheets of art paper (by the 1000's).

Maximize

$$250x + 200y + 400z$$

subject to

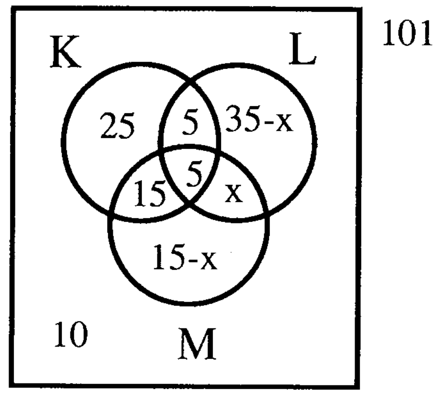
$$\begin{aligned} 0.7x + 1.5y + 0.5z &\leq 14 \\ 5x + 6y + 7z &\leq 250 \\ 300x + 500y + 600z &\leq 7000. \end{aligned}$$

and the implicit constraints $x, y, z, \geq 0$.

KEY
FINAL – VERSION 1

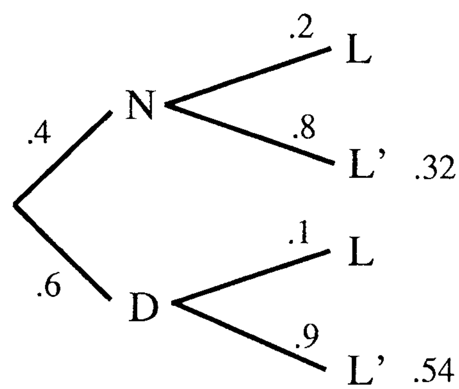
1. B

Use the Venn diagram.



2. B

From the data we obtain a tree diagram.



Therefore $\Pr[N|L'] = \frac{\Pr[N \cap L']}{\Pr[L']} = \frac{.32}{.32+.54} = .372 = \frac{16}{43}$.

3. C

Compute the stable probability vector; *i.e.* solve the system W(P – I) = 0 where $W = [\, w_1 \quad w_2 \,]$, P is the given transition matrix, and I is the identity matrix. The long run probability of being in state 2 is w_2.

4. B

Since all of the w_i's must sum to 1, we have $w_4 = 1 - \frac{3}{8} - \frac{2}{8} - \frac{2}{8} = \frac{1}{8}$. Therefore $\Pr[E] = w_2 + w_4 = \frac{3}{8}$.

5. B

$E \cap F'$ and $E \cap F$ partition E. Therefore $Pr[E] = Pr[E \cap F'] + Pr[E \cap F]$. Since E and F are independent, we may replace $Pr[E \cap F]$ with $Pr[E]\,Pr[F]$. Substituting known quantities we obtain $.3 = Pr[E \cap F'] + (.3)(.4)$. Therefore $Pr[E \cap F'] = .18$.

6. A

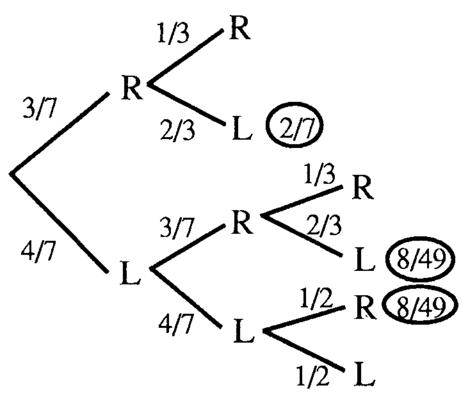
Partition the collection of all committees into three sets: those with no Democrats, those with one Democrat and those with at least two Democrats. There are C(8,4) possible committees without the restriction "at least two Democrats." There are C(4,4) possible committees with no Democrats. There are $4 \cdot C(4,3)$ committees with one Democrat. Therefore, by the partition principle, the number of committees with at least two Democrats is $C(8,4) - C(4,4) - 4 \cdot C(4,3) = 53$.

7. B

There are 36 equally likely outcomes (by multiplication principle). Of these, 14 sum to 5, 7 or 9. Therefore, the probability of not summing to 5, 7 or 9 is $1 - \frac{14}{36} = \frac{11}{18}$.

8. C

Use a tree diagram.



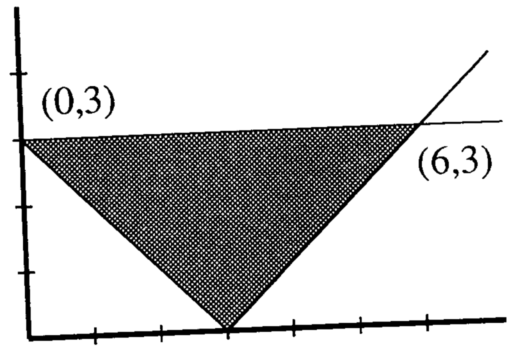
9. D

Compute.

10. C

Compute.

11. D Evaluate the objective function at the corner points of the feasible set graphed below.



12. D Let

$$x = \# \text{ of small kites,}$$
$$y = \# \text{ of large kites.}$$

Then

$$4x + 8y = 160$$
$$2x + 3y = 70.$$

From the first equation (solving for x) we obtain $x = 40 - 2y$. Substituting into the second and solving for y: $2(40 - 2y) + 3y = 70$ which reduces to $y = 10$.

13. D Compute the reduced row echelon form.

14. A Compute the reduced row echelon form.

15. C Compute the reduced row echelon form.

16. D These lines all pass through the point (-3,5).

17. B $X = A^{-1}B$, so $y = 2$.

18. D

Flipping a coin and counting "heads" is a Bernoulli trial. The probability density function of X is

x_j	$\Pr[X = x_j]$
3	1/8
4	3/8
5	3/8
6	1/8

The expected value is

$$3 \cdot \frac{1}{8} + 4 \cdot \frac{3}{8} + 5 \cdot \frac{3}{8} + 6 \cdot \frac{1}{8} = 4.5$$

19. D

20. D

21. A

First note that the fourth power of the matrix has all positive entries, thus it is regular. Next compute the stable vector by solving the matrix equation $W(P - I) = 0$ where W is the (unknown) stable vector, P is the given transition matrix and I is the identity matrix.

22. A

Check each point.

23. A

Let

$$x = \text{ number of small packages,}$$
$$y = \text{number of large packages.}$$

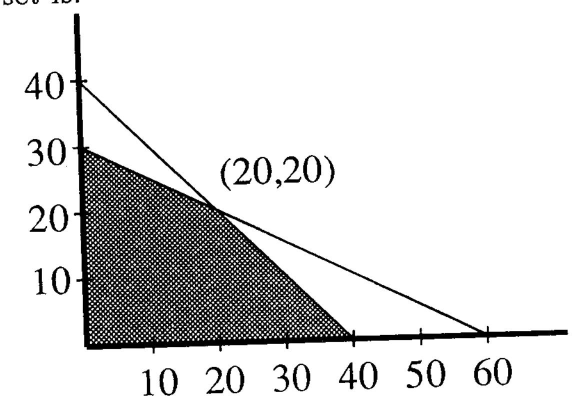
Then the linear programming problem is to maximize

$$x + 3y$$

subject to the constraints

$$x \geq 0$$
$$y \geq 0$$
$$4x + 8y \leq 240$$
$$4x + 4y \leq 160.$$

The feasible set is:



24. B

(I – A)X = D.

25. B

Let E denote the event "exactly two heads" and F denote "at least one head." Then E ∩ F = E. Thus $\Pr[E|F] = \frac{\Pr[E \cap F]}{\Pr[F]} = \frac{\Pr[E]}{\Pr[F]}$. Using Bernoulli trials compute: $\Pr[E] = C(4,2) \cdot \frac{1}{2}^4 = 6 \cdot \frac{1}{2}^4$ and $\Pr[F] = 1 - C(4,0) \cdot \frac{1}{2}^4 = (2^4 - 1)(\frac{1}{2}^4)$. Thus $\Pr[E|F] = \frac{2}{5}$.

26. D

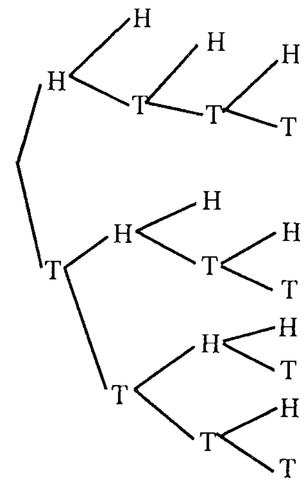
$E' \cap F' = (E \cup F)'$. Since $n(U) = 40$, $n((E \cup F)') = 40 - n(E \cup F)$. $n(E \cup F) = n(E) + n(F) - n(E \cap F) = 10 + 17 - 6 = 21$. Therefore $n(E' \cap F') = 40 - 21 = 19$.

27. D

Multiplication principle.

28. D

Use a tree diagram.

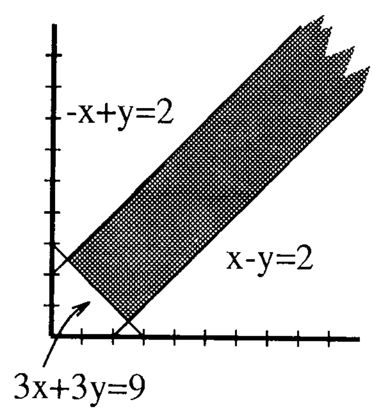


29. D

The position of Paris on the itenerary must be chosen (5 possibilities). The other four cities must be chosen in order (P(5,4) possibilities). By multiplication principle there are $5 \cdot P(5,4) = 5 \cdot 5!$ possible iteneraries.

30. B

The graph of the feasible set is given below.



31. A

Let L denote "likes linear programming" and P denote "likes probability." $L \cap P$ and $L \cap P'$ partition L, so $n(L \cap P') + n(L \cap P) = n(L)$. Substituting known quantities we obtain $n(L \cap P') + 20 = 60$. Therefore $n(L \cap P') = 40$.

32. B

33. C

The answer is given by the (2,2) entry of the 2-step transition matrix.

KEY
FINAL – VERSION 2

1. C	2. E	3. F	4. A	5. D
6. D	7. H	8. D	9. B	10. G
11. A	12. G	13. B	14. C	15. H
16. F	17. D	18. A	19. B	20. E
21. F	22. E	23. H	24. G	25. C

KEY
QUIZ 1

1. a) $\{2, 4\}$ b) $\{a, b, c, d, 1\}$ c) $\{1, 3, 5, a\}$ d) $\{b\}$
2. a) 975   b) 675

KEY
QUIZ 2

a) 15   b) 15^{40} c) 14^{40}

KEY
QUIZ 3

1. 0.58   2. 0.44   3. 0.8

KEY
QUIZ 4

1. a) 0.120   b) 0.125
2. a) $C(8,4)(\frac{1}{3})^4(\frac{2}{3})^4$ b) $1-(\frac{2}{3})^8$

KEY
QUIZ 5

a)

x_j	$\Pr[X = x_j]$
8	5/36
9	4/36
10	3/36
-3	24/36

b) 17/18

KEY
QUIZ 6

1. a) $y = -4x + 9$ b) $y = -4x + 7$
2. 100 standard boxes and 0 heavy-duty boxes

KEY
QUIZ 7

a)

$$\left[\begin{array}{ccc|c} 0 & -2 & 1 & 3 \\ 1 & -10 & 5 & 14 \\ 1 & -6 & 3 & 8 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & -\frac{1}{2} & -\frac{3}{2} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

b) $x = -1 \quad y = -\frac{3}{2} + \frac{1}{2}z \quad z$ arbitrary

KEY
QUIZ 8

$$\begin{bmatrix} 5 & 0 & -7 \\ 0 & 1 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$

KEY QUIZ 9

1. $A = \begin{bmatrix} .4 & .5 \\ .3 & .6 \end{bmatrix}$ 2. $(I - A)^{-1} = \begin{bmatrix} 40/9 & 50/9 \\ 10/3 & 20/3 \end{bmatrix}$ 3. $\begin{bmatrix} 650 \\ 600 \end{bmatrix}$

KEY QUIZ 10

Let

$$x = \text{acres of fish A}$$
$$y = \text{acres of fish B}$$
$$z = \text{acres of fish C}$$

Maximize $295x + 265y + 225z$ subject to

$$x \geq 0$$
$$y \geq 0$$
$$z \geq 0$$
$$x + 3y + z \leq 260$$
$$75x + 65y + 55z \leq 75000$$
$$x + y + z \leq 170.$$

KEY QUIZ 11

a) There is no solution. b) The minimum is -14; it occurs at (-2,-4).

KEY QUIZ 12

$p_{11}(2) = 0.25$ $p_{12}(2) = 0.33$ $p_{13}(2) = 0.42$

KEY QUIZ 13

1. This Markov chain is regular because P^2 has all positive entries.
2. $W = [\frac{2}{7} \quad \frac{3}{7} \quad \frac{3}{7}]$