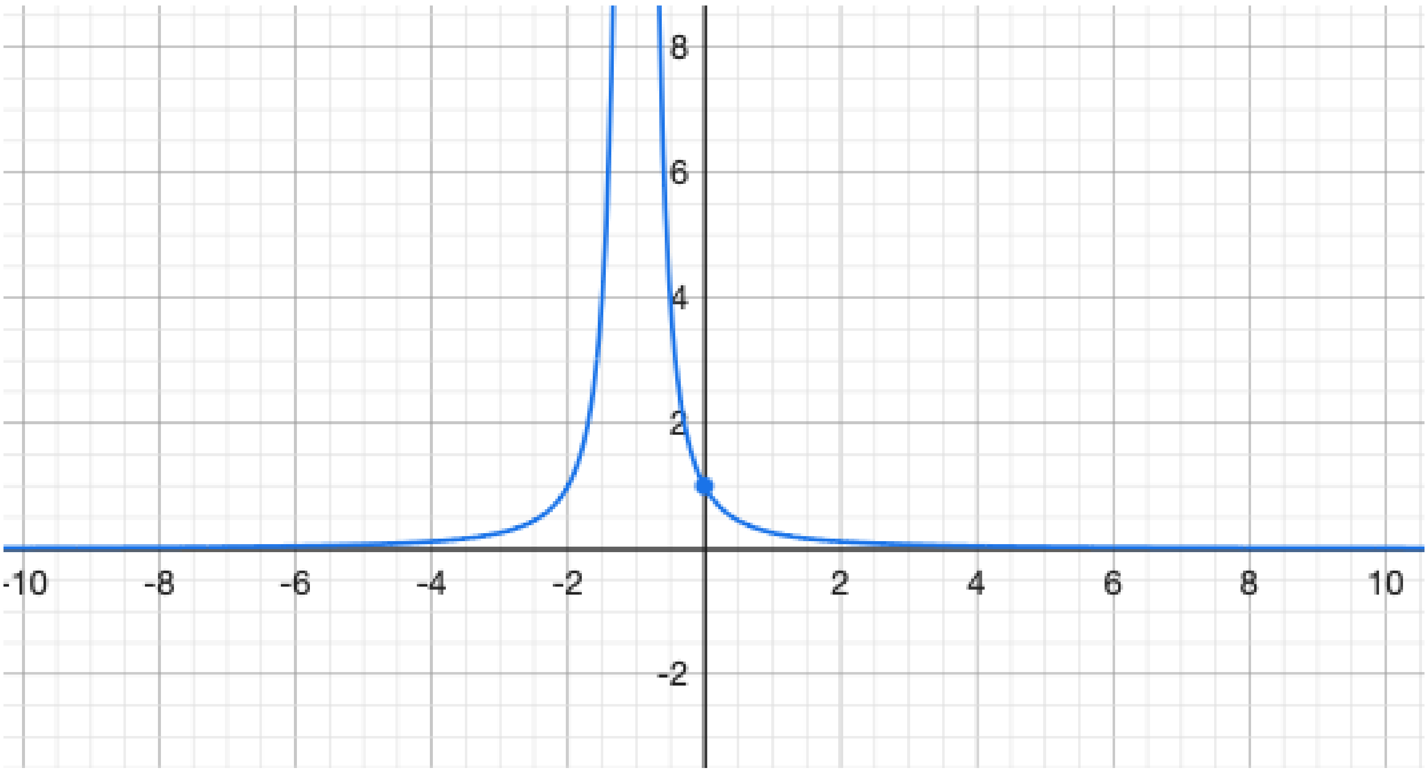
213 Problem Set 1 Solutions

Points to remember ... you needed to complete 3 problems of your own that have answers in the book. I'm just checking that they are there and complete. It's your job to check those. From the syllabus "Each assignment will be counted in the following manner: the exercises will be checked for completeness and will be worth half of the credit on the assignment. The remaining problems will be scored out of four points each". So, the problem set will be out of 8+8=16 points.

§1.7 7. $f(x) = x^2 + 2x - 3$, $h(x) = \frac{1}{x+4}$, so $h \circ f(x) = \frac{1}{(x^2+2x-3)+4} = \frac{1}{x^2+2x+1} = \frac{1}{(x+1)^2}$. The only potential problem with domain here is division by zero. The denominator equals zero at $x = -1$. That is the only place $h \circ f$ is undefined. So, the domain is all real numbers except -1, i.e. $\{x \mid x \neq -1\}$ i.e. $(-\infty, -1) \cup (-1, \infty)$. Again looking at the final form, notice that the denominator is squared, so it will never be negative. Furthermore, one divided by a number will never be zero. Therefore the range for this function is all positive reals, i.e. $(0, \infty)$. If we look at the graph, we can see hints of all of this. We notice that there is a gap at $x = -1$, and that all the values are above the y-axis.



§1.7 15. This one should be shorter ... solve $5^x = 16$ for x. To undo exponentials we use logarithms. I would commonly use the natural logarithm for everything. I take the natural logarithm of both sides $\ln 5^x = \ln 16$. Then I use the property that the exponent can come down in front of a logarithm: $\log a^p = p \log a$, to get $x \ln 5 = \ln 16$. Finally I can divide both sides by the number $\ln 5$ to get $x = \frac{\ln 16}{\ln 5}$. If you use most other logarithms, say base b, it works out the same: $\log_b 5^x = \log_b 16$, $x \log_b 5 = \log_b 16$, and $x = \frac{\log_b 16}{\log_b 5}$. But, there is one special base that makes it work out quite nicely and easily. If you use a base of 5, we have the special property that logs and exponentials cancel, so $\log_5 5^x = x$. This makes short work of the question: $\log_5 5^x = \log_5 16$, so $x = \log_5 16$. These answers all look different (the first is the second when $b = e$). They are all equal.