222 §7.2

Let's start with wrap up from Lab 15 and gather some ideas about $M(x)$:

(1) What is $M(1)$?

(2) What is the sign of $M(x)$ for $x > 1$?

(3) What happens to $M(x)$ as x increases?

(4) What is the sign of $M(.5)$?

(5) After noticing the answer to (4), what is the domain of $M(x)$?

(6) What is $\lim_{x\to 0^+} M(x)$?

(7) What is $M(1.5) + M(2)$ close to?

(8) What are some other sums that you can check like (7)?

(9) What does $M(a) + M(b)$ appear to equal? [Do you have a proof?]

(10) What is $\frac{d}{dx}M(x)$? [Do you have a proof?]

(11) What does $M(\frac{a}{b})$ equal? [Do you have a proof?]

(12) What does $M(a^r)$ equal? [Do you have a proof for all the different cases?]

(13) So, what function is $M(x)$?

(14) And hence, what is $\int \frac{1}{x}dx$?

(15) What is $\int \frac{x}{x^2+1}dx$? [This is for practice]

(16) What is $\int \tan(x)dx$? [This is worth knowing]

(17) What is $\frac{d}{dx}\ln(kx)$? Is it interesting? Surprising?

One more thing for today. How would you find the derivative of $y = x^x$? The power rule doesn't apply. If you don't know what to do, the right first step is to try to definition. (18) What does the definition of derivative say for $y = x^x$? Does it help? Probably not much. One last new idea for today - a way for finding the derivative of complicated combinations of multiplication, division, and exponents. Taking logarithms of such combinations simplifies them, so maybe we can use it for finding derivatives. Let's try in this example. $\ln y = \ln x^x$. (19) What does the right side equal? Now let's take derivatives (implicitly). (20) What is the derivative of the left side (don't forget the chain rule and that y is a function of x)? (21) What is the derivative of the right side? (22) Solve this equation for $\frac{dy}{dx}$. As a last step, substitute x^x back in for y. This should yield $\frac{dy}{dx} = x^x(\ln x + 1)$, which really could not have been obtained using other means. The process of using logarithms to take derivatives is called (rather uncreatively but also informatively) *logarithmic differentiation*.