Up to now we've done some creative algebra - we surely have plenty more to come. Today is our day of creative trigonometry. To be able to do creative trigonometry - that is rewriting expressions to suit your needs - you need to be fluent in some trigonometry identities. Before we begin with calculus, let's gather some identities. It's a good thing that the most important identity is the one that people know best. (1) What is that? We get two more for free from that one - (2) divide by $\cos^2\theta$ and $\sin^2\theta$ to get two more identities. So far that's pretty painless. Now, we need two more, and if you don't know them, it's probably at this point best to just look them up. Actually if you knew either $\sin(\alpha+\beta)$ or $\cos(\alpha+\beta)$ that would be great, because you could get all the others from there, despite the fact that we don't have much direct use for them in calculus. (3) Somehow you need to get to both $\sin(2\theta)$ and $\cos(2\theta)$. The second one has several forms. (4) Use the identity that everyone knows to write it $\cos(2\theta)$ in three difference ways. Once where the right hand side only has $\sin\theta$, once where it only has $\cos\theta$ and once where it has both. (5) Take the one that only has $\sin\theta$ and solve for $\sin^2\theta$. Do the same with the one that only has $\cos\theta$.

After all of the above, you should make sure that you have gathered the following identities:

$$\sin^2\theta+\cdots, \sec^2\theta=\cdots, \csc^2\theta=\cdots, \sin(2\theta)=\cdots, \cos(2\theta)=\cdots=\cdots=\cdots, \cos^2\theta=\cdots, \sin^2\theta=\cdots$$

The last two should feature $\cos(2\theta)$ and no other trigonometry functions. And that's what we'll need. Now to calculus.

Let's start with a good example for many reasons. (6) Integrate $\int \sin x\cos x dx$. Did you use $u=\sin x$? I'm guessing that's most popular. (7) Try with $u=\cos x$ and then try a third time using the $\sin 2x$ identity from above. (8) How do your three answers compare?

What we're really working on in this section is integration of powers and products of trigonometry. Let's continue the above example with two that should follow easily (9) Integrate $\int \sin^7 x\cos x dx$ and $\int \sin x\cos^{63} x dx$. Fine. Now what about $\int \cos^3 x dx$? This doesn't work as nicely, but there's a nice move to make it better - here's the creative trigonometry. What if we use $\cos^2 x = 1-\sin^2 x$? Then $\int \cos^3 x dx = \int (1-\sin^2 x)\cos x dx$. (10) Finish this integral. Continuing with question 4 from lab 17: what about $\int \cos^5 x dx$? (11) Finish this integral by using the same identity. (12) What about $\int \cos^4 x\sin^3 x dx$?

This question in lab finishes with $\int \cos^2 x dx$. This doesn't work the same way. (13) Why not? Above we have an identity for $\cos^2 x$. (14) Make that substitution and then integrate. Saved by the idenity! What about $\int \cos^4 x dx$? (15) Substitute and square, but then you will find a $\cos^2(2x)$ which is a new problem, so (16) substitute again. Finally integrate. Ask me if there are questions about that. With some extensions that's about the whole story for any integral like $\int \sin^p x\cos^q x dx$.

What about some other trigonometry functions? Consider $\int \sec^2 x\tan^2 x dx$. (17) What makes this question like our first questions with sine and cosine? (18) Finish it. (19) then do $\int \sec^4 x\tan^2 x dx$ and $\int \tan^5 x\sec^3 x dx$.

We discussed $\int \tan x dx$ in §7.2. What about $\int \sec x dx$? This is usually done with a perhaps surprising move that is done because it works. (20) Try multiplying by $\frac{\sec x+\tan x}{\sec x+\tan x}$, see what you get and then integrate.

If the functions don't seem to go together, like $\int \tan x \sin x dx$ it is best to (21) convert to sines and cosines and then integrate. This is a good suggestion in any case when nothing else seems to work: (22) $\int \tan x \sec^4 x dx$.

I've skipped integrals with $\csc x$ and $\cot x$, mostly because they are nearly identical to those with $\tan x$ and $\sec x$. Ask me if you want to discuss them in class.