222 §8.6

A lot of this needs to be wrapping up lab 18. (1) Draw a generic looking function (like the one in lab - rising and falling) from $x = a$ to $x = b$. Divide it into some equal base subintervals (like the picture in lab). Make it an even number of subintervals. Make a total of five copies of this picture. (2) On the first copy, draw in rectangles using left endpoints of each subinterval to determine the height of the rectangle. (3) On the second copy, draw in rectangles using right endpoints. (4) On the third copy, draw in rectangles using the midpoints of the subintervals to determine the heights. (5) On the fourth copy, draw in trapezoids by connecting the left endpoint on the curve to the right endpoint on the curve. These are our left, right, midpoint, and trapezoid approximations. We'll come back to the fifth copy soon. Compare how each of these methods seem to do at approximating the area. Notice that the left endpoint approximation does not use the value of the function at $x = b$ And similarly, the right endpoint approximation does not use the value of the function a $x = a$. Note that the trapezoid approximation is the average of the left and the right. Note that the trapezoid and midpoint approximations look pretty similar. Make sure that you can draw pictures like this for all of our approximations. Make sure that you can compute them by hand for a small number of subdivisions, and make sure that you can compute them by calculator for large numbers.

In lab she says "The details are messy, but the area under the parabola through $(x_0, f(x_0))$, $(x_1, f(x_1))$ and $(x_2, (f(x_2))$ can be shown to be $\frac{\Delta x}{3}[f(x_0)+4f(x_1)+f(x_2)]$, where $\Delta x = \frac{b-a}{4} = \frac{x_4-x_0}{4}$." Let's look at some of those messy details. Instead of x_0, x_1, x_2 we will shift the middle to 0 and use $-h, 0, h$. This makes computation easier, but notice shifting horizontally doesn't affect the area.

We're looking for the area under a parabola. Because it's a parabola, it can be written as $f(x) = Ax^2 + Bx + C$ for some A, B, C. Integrate to find the area under this curve from $-h$ to h. Since we expect $\frac{\Delta x}{3}$ in front, please factor out $\frac{h}{3}$.

What are $f(-h), f(0), f(h)$? Compute $f(-h) + 4f(0) + f(h)$. Compare this to what you found in the integral. They should be the same. As in lab, we then add up segments to get the formula you found there. Make sure you have it. That wasn't too messy, but it was also made easier by knowing what answer we were heading for.