No notable wrap-up from Lab 21, aside from the obvious - make sure you know the limit comparison test.

222 §10.5 In fact, I want to go back to Lab 20, instead. There we focused on geometric series. There I hope we found that $\sum_{n=0}^{\infty} ar^n$ converges when $|r| < 1$ and diverges otherwise. Now, we want to look at some ways to use this information. Let's look at a simple example, just a sequence really

$$1596, 27132, 461244, 7841148, 133299516, \ldots$$

In this geometric sequence, (1) how do you find what the ratio, r, equals? You've know this for years. And it still works as well. What if we have a geometric sequence

$$t_0, t_1, t_2, t_3, t_4, t_5, t_6, \ldots$$

(2) how do you find the ratio? (3) Do you have more than one option? (4) Which two terms matter most? That's a bit of a trick question. The answer is "the last two", which should mean we need to take the limit. This is consistent with out neverending preference for the end of a series. Let's look at a simple example: $\sum_{n=1}^{\infty} \frac{n^{10}}{2^n}$. It would be great if this looks familiar - we have seen it in Lab 19 2b. There we saw that $\lim_{n\to\infty} \frac{n^{10}}{2^n} = 0$, so it could possibly converge.

So, we need to do some more work. The "last" ratio can be seen to be $r = \lim_{n\to\infty} \frac{t_{n+1}}{t_n}$. (5) Compute this limit for this example. I hope you get $r = \lim_{n\to\infty} \frac{\frac{(n+1)^{10}}{2^{n+1}}}{\frac{n^{10}}{2^n}} = \lim_{n\to\infty} \frac{(n+1)^{10}2^n}{n^{10}2^{n+1}} = \lim_{n\to\infty} \frac{1}{2}\left(\frac{n+1}{n}\right)^{10} = \frac{1}{2}$. Now, recall our work with geometric series. (6) For which values of r does a geometric sequence converge? (7) So, what can you say about the convergence of $\sum_{n=1}^{\infty} \frac{n^{10}}{2^n}$?

Let's look at a couple more examples. What about $\sum_{n=0}^{\infty} \frac{10^n}{n!}$. (8) Complete the work to find $r = \lim_{n\to\infty} \frac{10}{n+1}$. In particular make sure you are careful with the factorials. (9) What can you say about the convergence of $\sum_{n=0}^{\infty} \frac{10^n}{n!}$? (10) How important was the number 10 in this example? (11) What can you say about the convergence of $\sum_{n=0}^{\infty} \frac{x^n}{n!}$? This is an important example that we started chapter 10 with. Remember when we found a series for e^x in Lab 22? We have now, finally, shown that this series converges for *any* x. This is a *big deal*.

The next class of examples illustrates an important point. Remember our p-series? They were $\sum_{n=1}^{\infty} \frac{1}{n^p}$. (12) When do they converge? Now apply the ratio test to these series. (13) What do you get as a ratio r? (14) Does your ratio depend on p? As I hope you know, the ratio $|r| = 1$ is the boundary for convergence for geometric series. What the examples of p-series tells us is that for the ratio test $|r| = 1$ is too close to call.

In class I want to look at the example $\sum_{n=1}^{\infty} \frac{n!}{n^n}$. Think about it ahead if you want.

One more idea here. Going back to our geometric sequence, $t_n = ar^n$. (15) Solve this equation for r. We will take a slightly different approach using limits and the fact that no matter what a equals (ok, presuming it's not zero, but that would be silly), $\lim\limits_{n\to\infty} \sqrt[n]{a} = 1$. Using this, we can find r without needing to know a, $r = \lim\limits_{n\to\infty} \sqrt[n]{t_n}$.

This turns out to not be the most useful idea (that might go to the aforementioned ratio test), but there are some examples where the root test is just the thing we need. Consider $\sum\limits_{n=3}^{\infty} \frac{1}{\ln^n n}$. (16) Try using the ratio test enough to see that it is difficult. (17) Now try using the root test and quickly see how easy it is. We get $r = 0$, and so (18) what can you say about convergence?

Two more examples to consider for class. (19) Try to test $\sum\limits_{n=1}^{\infty} \frac{n}{(n^2+6)^n}$. And (20) show that the root test yields $r = 1$ for all p-series, thus showing that again $|r| = 1$ is not giving a decisive result.