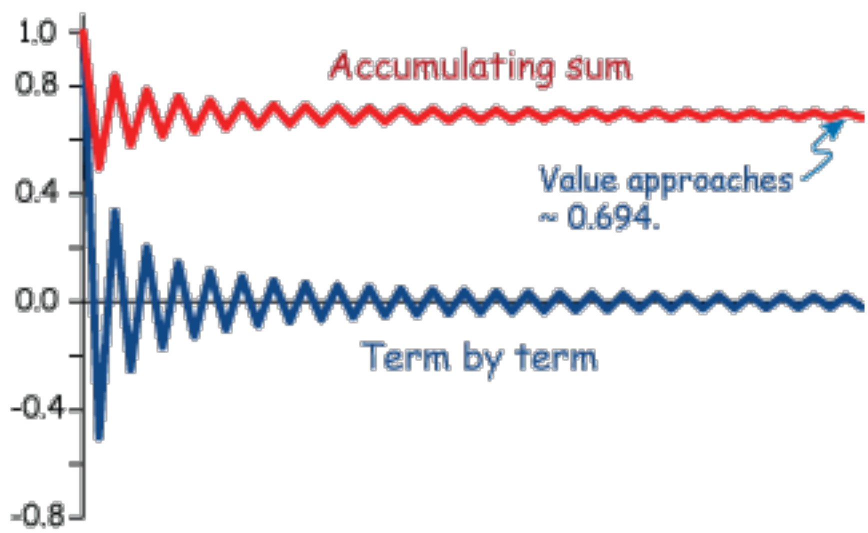
222 §10.6 We saw a little bit of alternating series in Lab 20. In fact, although we have repeatedly focused on series with all positive terms (in the integral test, and the comparison tests), alternating series are even more special than all positive series. The fact that the series alternates between positive and negative terms gives us a good idea for what is going on.

Let's return to the example from lab, the alternating harmonic series:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$$



Notice a few things. The simplest one to see is that the partial sums alternate between moving up and down, based on whether the next term is positive or negative. Second, because this is the alternating *harmonic* series and $\frac{1}{n}$ is decreasing, the successive jumps are getting smaller each time. Third, because $\lim_{n\to\infty} \frac{1}{n} = 0$, the jumps are settling down and the limit can (and does) converge. [Why does it converge? Because the positive ones are a decreasing bounded sequence, and the increasing ones are an increasing bounded sequence, and since the terms are going to zero, the two convergent sequences are settling to the same limit.]

(1) What are the two important features of this series that makes it converge? (2) What goes wrong if the sequence is not decreasing? (3) What goes wrong if the sequence does not approach zero?

Here are two variations on our first example. (4) What can you say about $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$? (5) What are you justifications?

One more useful thing about convergent alternating series. Since the partial sums jump back and forth over the limit, any two of them will provide bounds. (6) Using your calculator (as in Lab 20, take the first 100 and then first 101 terms to get upper and lower bounds for the sum of this series.

(7) What can you say about $\sum_{n=1}^{\infty} (-1)^n \frac{1}{1+1/n}$? (8) Why is this one different?

Going back to the alternating harmonic series, it is interesting because it converges although the non-alternating version diverges. On the other hand, consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$. Notice here that the non-alternating version also converges.

If $\sum_{n=1}^{\infty}(-1)^n a_n$ converges and $\sum_{n=1}^{\infty}|(-1)^n a_n|$ also converges we say that $\sum_{n=1}^{\infty}(-1)^n a_n$ converges *absolutely* (i.e. with absolute values). In the other case, if $\sum_{n=1}^{\infty}(-1)^n a_n$ converges but $\sum_{n=1}^{\infty}|(-1)^n a_n|$ does not we say that $\sum_{n=1}^{\infty}(-1)^n a_n$ *conditionally* converges (i.e. conditional on the absolute values not being there).

(9) Give examples from this discussion of series that are conditionally convergent, and examples that are absolutely convergent. In class we will see some curious properties of conditionally convergent series that are hinted at at the end of lab 20.