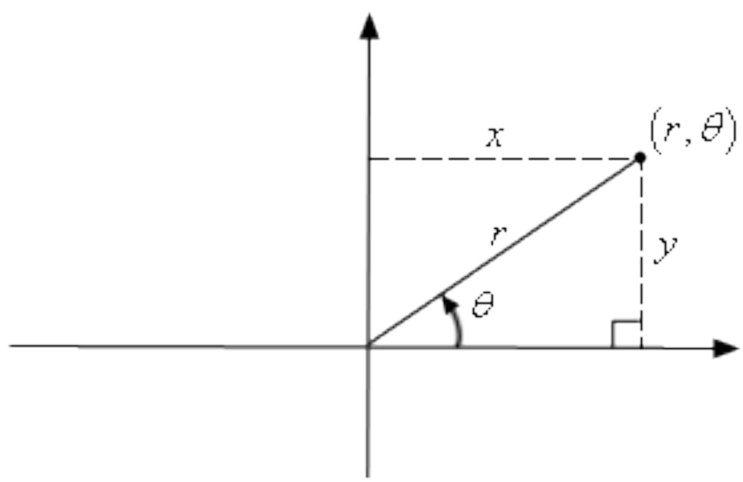
Welcome to chapter 11. It's our last chapter. It's the calculus of pretty pictures. You know from high school that there are limitations to what is possible for $y = f(x)$ functions. But, what if the path you travel doesn't fit with those limitations? Other things can and do happen. In particular there are paths that are traversed in time that wind about and cross themselves. How do we represent them? The first answer (although there's a more specialised one later) is something called parametric equations. Any path in the plane can be written with *both* x and y as functions of time, just by naming the coordinates for any given time. The most important curve that cannot be written as a $y = f(x)$ function is a circle. If we let the time t be the same as the angle around the unit circle (remember this means $r = 1$) centered at the origin with the positive x-axis being the angle 0, then we have an important right-triangle picture:



(1) From this picture, what is $x(t)$ (x in terms of t)? (2) What is $y(t)$? What range of values for t (or θ) give the entire circle, once around [this is calculus]? Together this gives us a *parametrisation* of the standard unit circle:

$$(\cos t, \sin t), 0 \leq t \leq 2\pi \text{ or } x = \cos t, y = \sin t$$

One thing that is particularly nice about parametric equations is that they are quite easy to transform. (3) If we wish to change this to a generic circle with centre (a, b) of radius r, what is a pair of transformations that will easily do this? Be careful about the order. (4) How does performing the first transformation change the equations? (5) How does performing the second change the equations? (6) What is a parametrisation of the circle with centre (a, b) of radius r?

Another important detail - these are examples of parametrisations, but they are not unique. Here are some other parametrisations of the standard unit circle:

$$(\cos t^2, \sin t^2), 0 \leq t \leq \sqrt{2\pi}; (\cos t, \sin t), 2\pi \leq t \leq 4\pi$$

There are other parametrisation that produce the same curve, but with differences. (7) What are two things that are different about $(\sin t, \cos t), 0 \leq t \leq 2\pi$?

Ok, let's end with a famous example, that we'll return to in §11.2. The cycloid is a curve with lots of fascinating properties. It is the shape that wires hang in (ok, upside down), it is the curve with the strange property that wherever you start on it, it takes the same time to reach the bottom (due to gravity). It is also the curve that has the fastest rate of descent between two given not vertical points (faster than a straight line). So, a crazy curve. The easiest way to define it is as follows . . . it is the path that a point on a wheel follows as the wheel spins. Consider that the point, P, starts at the bottom of the wheel. And suppose the wheel has radius 1. Furthermore we put some coordinates down so that the starting point is the origin, and the "road" is the x-axis. And the wheel is traveling to the right. Finally, assume the wheel is traveling at speed = 1. There's lots of assumptions. I think that's enough to be specific. (8) If the radius is 1, what is the y-coordinate of the *centre* of the wheel? (remember, it is sitting on the x-axis). Given this, and the speed, the centre of the wheel, at time t has coordinates $(t, 1)$. (9) As the wheel spins to the right, P moves in which direction(s)? (10) Well, that's not easy, but what if it ask the same question relative to the centre of the circle? (by this we mean, if the centre is not moving, and the wheel is just spinning, which directions does P move?) (11) Draw a picture with the wheel rotated thru some angle, t (make it less than a quarter turn so you can easily see). (12) How far is P below the centre point? (will need to refer to angle t) (13) How far is P to the left of the centre point? (again refers to angle t) (14) Combine all this information to write the coordinates of P. (i.e. start with the centre and adjust right and left in terms of t). In the end you should get $(t - \sin t, 1 - \cos t)$. We'll talk about this is if there are problems. If you see it already, congratulations - you have derived parametric equations for the cycloid, one of the most famous and fancy curves studied by mathematicians (which is not very friendly with function equations).