Instead of a wrap up from Lab 14, we have a different perspective, that may be easier to make sense of. If we travel from one cartesian coordinate point to another, we will change both x and y coordinates. Suppose the change in x is, unsurprisingly, Δx, and the change in y is Δy. (1) What is the change in distance between these two points, maybe call it Δs? If we move along a curve, like in Lab, breaking it up into little pieces, the total distance will be given by the sum of these Δs. Now, if we do this over time, each of the Δx and Δy depend on time. If we make the segments infinitely small, instead of adding we have an integral. And instead of Δx and Δy we have $\frac{dx}{dt}$ and $\frac{dy}{dt}$. (2) Check to see that making these changes leads to $\int_{t_0}^{t_1} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$.

To make sure you understand what this formula says, (3) use it to find the distance around a circle of radius r, i.e. the circumference, given by $x = r\cos t$, $y = r\sin t$ (what range of values should you use for t to get the whole circle?). This time I shouldn't have to tell you the answer – check that you get the answer you expect.

Now, to connect back to Lab 14, suppose $x = t$ and $y = f(x)$. (4) Substitute into the above formula for arclength to see that you get the same integral formula used in Lab 14. I believe remembering the parametrised version is best - it is most general and it also makes the most sense.

The unfortunate fact about arclength is that almost all the integrals that the formula produces are impossible to compute exactly. It's famous for such things. There was one example in lab, and we have the example above for the circle, but almost all others require approximations for the integrals. Fortunately we have plenty of those. So, for any practical use we are well-equipped with the formula.

Here is another, unrelated, instance of calculus with parametric curves. We have a common calculus interest in tangent lines. What if we seek to find tangent lines to parametric curves? Fortunately, the situation is not that bad. It arises from this fact: if we think of $x(t)$ and $y(t)$, but imagine y first as a function of x, i.e. $y(x(t))$, then we can take a derivative using the chain rule. (5) Please do so. In Leibniz notation you should get $\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$. (6) Please solve for $\frac{dy}{dx}$. That should be nice and easy to remember. This is what we use to find slopes of tangent lines to parametric curves.

For a simple beginning, (7) what is the slope of the tangent line to the circle above at time t? (8) When do we have a horizontal tangent? (9) When a vertical tangent?

If we go back to the cycloid $(t - \sin t, 1 - \cos t)$, (10) when does this curve have a horizontal tangent?

One more calculus idea . . . we can look at area under these parametric curves. You might not think about it, but to find area under a curve $y(x)$, you integrate $\int y dx$. If we think back to our circle, what is y? What is dx? Since we are looking for area *under* it's probably best to just use half of the circle and then multiply by two. (11) What range of t values gives half of the circle? At this point you should have the following integral $\int_0^\pi \sin t(-\sin t)dt$. (12) Revisit a little of chapter 8 to compute this integral (not a bad time to review - there is a final coming up). (13) Does the answer make sense? (14) Why not? Here's the important detail. (15) when we traverse from time $t = 0$ to time $t = \pi$, are we going right-to-left or left-to-right? (16) Which way gives us a positive integral (in Calc I)? Therefore we need to switch the limits to get the answer we expect. I hope it's all good now.

To try out one more, (17) find the area under one arch of the cycloid. In this case, we are traversing in the correct direction. That's comforting.