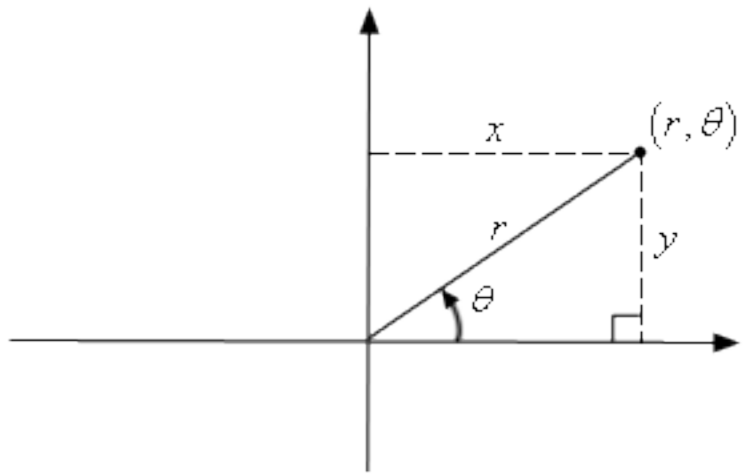
Here we will start thinking about a new coordinate system. This is technically a special case of parametrisation, but it is a very important and useful case. Why do we need new coordinates? What's wrong with the old ones? Well, they work out great if you're in Manhattan (really great, actually), but not as great if you're in the centre of a field (which is rather more likely out here), or the ocean, or at the South pole. At the pole, we should really be using *polar* coordinates. Instead of using number of blocks north and east, we will use direction and distance. Here's hoping I gave you some polar coordinate paper. And now let's start the same way you started with Cartesian coordinates - by plotting points.

Just like when plotting in Cartesian coordinates, the first step is to set our scale. With polar coordinates the angles aren't up to us, but we do have to set our length scale. Let's say the outer ring is distance 4 from the origin and scale our other points appropriately. Especially on this first time out, just to be sure that you see what is what, angle 0 from the centre is in what used to be the positive x-direction. (1) So, let's get started with plotting $(3, 0)$. The first number indicates the distance from the origin (usually named r) and the second the angle (usually named θ). Interestingly this is the same point as it was in rectangular coordinates. This won't last long. (2) Plot $(4, \frac{\pi}{2})$. (3) Plot $(2, \frac{5\pi}{4})$. Let's see a few little different things - (4) where is $(1\frac{1}{2}, -\frac{2\pi}{3})$? Ok, so how does that generalise - (5) what does having a negative angle mean? How about this - (6) plot $(0, \pi)$. (7) What is unusual there? Hint: (8) what happens when you plot $(0, \frac{\pi}{6})$? So, that's kinda curious - the origin is special.

How about something else different - (9) where is $(-1, \frac{\pi}{3})$? In case that isn't clear, check that $(-3, 0)$ is in the same place using polar or Cartesian coordinates. I hope you understood above that it is easy to give many names for the origin. But, any point has many names. Let's go back to $(2, \frac{5\pi}{4})$. (10) What are three other names for this same point? (11) What are two names with negative angles? (12) What are two names with negative r values? (13) What are two names with both negative and angles and negative r values? That's a long list of names. And, it's pretty clear there's nothing special about this point.

Let's wrap up some details here.

How are polar coordinates and Cartesian coordinates related? Draw a triangle on which you indicate x, y, r, and θ. I hope it looks like



Now, let's find some relationships. (14) Write an equation based on this picture that includes only x, y and r. Solve it for r. (15) Write a trigonometric function that equals something about x and r. Solve it for x. (16) Write a trigonometric function that equals something about y and r. Solve it for y. (17) Write a trigonometric function that equals something about x and y. Solve it for θ. Note that you should now have both r and θ in terms of x and y, and both x and y in terms of r and θ. This means you can convert back and forth between the two systems. This will come in handy from time to time.

After all this work on plotting points, what if we step up to equations? Starting with the simplest ones, compare to the simplest in Cartesian coordinates - (18) graph $y = 2$ and $x = -3$. Now, (19) plot $r = 2$ and $\theta = \frac{\pi}{3}$. To make sure everything is correct, (20) also plot $r = -3$ and $\theta = -\frac{3\pi}{4}$. (21) What is $r = 0$?