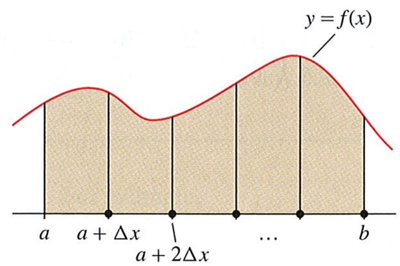
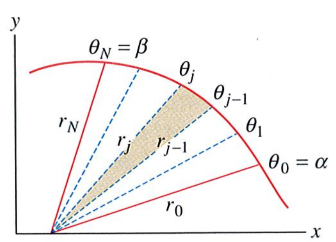
This is our very last section. (You probably knew that.) And we get one more visually compelling argument out of our polar coordinates. Filling in our calculus with polar coordinates, we have area left. To approach area we will think back to how we first did area in rectangular coordinates. Remember pictures like this:




The segments are not quite rectangles, but they become closer to rectangles the more we take, as if we take a limit we get an exact value using rectangles. So, we think with rectangles and then switch to an integral to get the exact answer. (1) If we think of $\Delta x$ as the width of the rectangles, what gives us the height of these rectangles for any $x$? (2) Once you know the width and the height, what is the area? Sum them to get the total area equal to $\sum f(x)\Delta x$ in terms of rectangles. (3) When we take a limit in order to transition to integrals, what integral gives us this area? (4) What is the range of $x$-values for this integral?

For our rectangular integrals, we think of the $x$ values being swept out left-to-right, not unlike an EKG or a seismograph. For polar coordinates we need a change of perspective. Now we will think of the $\theta$ values being swept out radially like the hands of a clock, but counterclockwise. This image is the main idea for today:




Because of the fact that the geometry is fundamentally different for polar coordinates, our formulas need to fundamentally change. In rectangular coordinates we divided the region into approximate rectangles. This time we will divide our region into approximate sectors of circles. Unlike with rectangles, you may not be familiar with the area formula for a sector of a circle. However, it's all elementary geometry. (5) What is the area of a circle of radius $r$? (6) If a sector has angle $\theta$ (radians, this is calculus), what fraction of the whole circle is the sector? (7) So, what is the area of a sector of a circle with radius $r$ and angle $\theta$?

Now that we've produced the basic geometry, back to our problem. We pick up with questions like (1) above: (8) If we think of $\Delta\theta$ as the angle for the sectors, what gives us the radius of these sectors for any $\theta$? (9) Once you know the angle and the radius, what is the area? Sum them to get the total area equal to $\sum \frac{1}{2} r(\theta)^2 \Delta\theta$ in terms of sectors. (10) When we take a limit in order to transition to integrals, what integral gives us this area? (11) What is the range of $\theta$-values for this integral? At this point you should have the formula $A = \int_{\theta_0}^{\theta_1} \frac{1}{2} r^2 d\theta$.

Ok, that's our background. (12) Check that the formula agrees with a known result - use it to compute the area of a circle of radius 4. In lab 24 you considered the three-petal graph of $r = \sin 3\theta$ and found the $\theta$ values that corresponded to the petal in the first quadrant. (13) Use those to find the area of that petal.

Now, we'll move to area between curves. (14) In Calc I how did you find area between curves? Notice that it comes from (Area under $f$) - (Area under $g$). For our polar work we thus have (Area inside $r_1$) - (Area inside $r_2$), and that this is definitely not the same as subtracting the radii. We got some set-up in our previous worksheet. Let's now use it. (15) Find the area between $r = 1$ and $r = 2\cos\theta$.

Last time we thought about the area between $r = \cos\theta$ and $r = \sin\theta$. This case is a little different because of questions about what curves to use. In fact, the answer changes. (16) What is the radial boundary from 0 to $\pi/4$? (This is the curve that you hit first when you travel out from the origin in that direction.) (17) What is the radial boundary from $\pi/4$ to $\pi/2$? Probably you want to compute this area by computing two separate integrals. (18) Do so.

The last example on the previous worksheet was looking at the area between $r = 2\cos\theta - 1$ and $r = 2$. This is even fancier. (19) See what you can do with it on your own, and then we'll see what we can do in class. Thank you for reading along all semester. I am grateful for your attention.