Mathematics 228 PS3 Solutions

6.1 26 Each year (presuming year is the time unit) the probability the island is occupied is halved. For n years it is occupied with probability $\left(\frac{1}{2}\right)^n$, and eventually it will be unoccupied and stay that way.

6.2 30 A Markov chain works well here. Two states, C and P for caterpilar or parasite controlling. 0.15 C $\rightarrow$ P and 0.03 P $\rightarrow$ C. We write our DTDS: $C_0 = 1$. $C_{t+1} = 0.85C_t + 0.03P_t = 0.85C_t + 0.03(1 - C_t)$.

So $C_{t+1} = 0.82C_t + 0.03$

We now compute: $C_0 = 1$, $C_1 = 0.85$, $C_2 = 0.727$, …, $C_{25} = 0.172503335647$ and we expect 8.62 to survive, much better.

6.2.54 Because of mutation we have the following Mendel square:

	A (0.99)	a (0.01)
A (0.99)	AA (0.9801)	Aa (0.0099)
a (0.01)	aA (0.0099)	aa (0.0001)

So Pr(homozygous) = 0.9801 + 0.0001 = 0.9802
and Pr(heterozygous) = 0.0198

6.3 38 First the bilogically expected case:
suppose alleles B and b are equally likely.
Each offspring may be BB, Bb, or bb with 1/4, 1/2, 1/4 probability.
We have the following possible pairs:

pair	probability
BB, BB	$\frac{1}{16}$
BB, Bb	$\frac{1}{4}$
BB, bb	$\frac{1}{8}$
Bb, Bb	$\frac{1}{4}$
Bb, bb	$\frac{1}{4}$
bb, bb	$\frac{1}{16}$

Here's a mathematical possibility: suppose allele B has probability 0.3 and allele b has probability 0.7. Each offspring may be BB, Bb, or bb with 0.09, .042, and 0.49 probability. We have the following possible pairs:

pair	probability
BB, BB	0.0081
BB, Bb	0.0756
BB, bb	0.0882
Bb, Bb	0.1764
Bb, bb	0.4116
bb, bb	0.2401

6.4 40 Draw a tree diagram as follows. First branch D and N with probabilities 0.8 and 0.2. From D branch +/– with probabilities 0.95 and 0.05. From N branch +/– with probabilities 0.1 and 0.9. We seek $\Pr(D|+)$ and $\Pr(D|-)$. To find these we need $\Pr(+)$, $\Pr(-)$, $\Pr(D\cap+)$ and $\Pr(D\cap-)$. The last two are simply computed from the tree diagram to be 0.8*0.95 = 0.76 and 0.8*0.05 = 0.04. To find $\Pr(+)$ we need the other case, which is 0.2*0.1 = 0.02. So $\Pr(+)$ = 0.76 + 0.02 = 0.78. Similarly we need another case for $\Pr(-)$ which is 0.2*0.9 = 0.18. So $\Pr(-)$ = 0.04 + 0.18 = 0.22. From this $\Pr(D|+) = 0.76/0.78 = \frac{38}{39}$ and $\Pr(D|-) = 0.04/0.22 = \frac{2}{11}$.

6.4.44 If you pick only tall offspring for the parents, the probability among the tall of BB is $\frac{1}{3}$ and the probabilty among the tall of Bb is $\frac{2}{3}$. From there and some more typical genetics we get the tree distributed in class. Probability that the plant is tall is $\frac{8}{9}$. Probability that the plant is Bb is $\frac{4}{9}$, so $\Pr(Bb \mid tall) = \frac{1}{2}$. See handout.

6.5 46 There are 19 transitions in the table (the last time doesn't transition so doesn't contribute). Of the 9 starting in, 3 leave, the rest stay in. Of the 10 starting out, 3 leave, the rest stay out. So we have $p_{t+1} = \frac{2}{3}p_t + \frac{3}{10}.(1 - p_t) = \frac{11}{30}p_t + \frac{3}{10}$. The equilibrium probability is $\frac{9}{19}$. The fraction actually in was $\frac{1}{2}$. Not identical values, but clearly related.

6.6 44 I will do this as a tree diagram. See handout. From there we combine the like colours. $\Pr(R) = 0.475$, $\Pr(B) = 0.3125$, $\Pr(G) = 0.2125$

6.7 34 Assuming the data is per year . . Let I be the number if immigrants that year. Here is our random variable

I	p
–10	0.4
0	0.2
1	0.3
2	0.1

$E(I) = -10(0.4) + 0 + 1(0.3) + 2(0.1) = -3.5$

In ten years, 35 people will leave. The population will not grow. The expectation may or may not seem close to the middle, depending on what 'middle' means to you.

6.7 42 Total mass = $\int_0^2 x^2(2 - x)dx = \frac{4}{3}$. So, now we'll use $\frac{3}{4}x^2(2 - x)$ as our close to pdf. So, to find the expected value / centre of mass we compute: $\int_0^2 x\frac{3}{4}x^2(2 - x)dx = 1\frac{1}{5}$, right of centre. $\rho(x)$ is maximal when $\rho'(x) = 0$ (because it equals zero at both endpoints). This happens when $x = \frac{4}{3}$ (and 0, but that's not it). Anyway, $\frac{4}{3}$ is close but not $\frac{6}{5}$.

6.8 40,44

R	p
5	0.25
0.25	0.25
1	0.5

The arithmetic mean is $1.8125 = \frac{1}{4}(5 + 0.25 + 1 + 1) = 5\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + \frac{1}{2}$

The geometric mean is $\sqrt[4]{5\left(\frac{1}{4}\right)1(1)} = 1.05737126344$

Or $e^{\frac{1}{4}(\ln 5 - \ln 4)}$ is the same value. This is > 1, so the population is growing. We expect that after 50 generations the population will be $100(1.05737126344)^{50}$ or about 1627, that's some exponential growth for you.

6.9 44

N_1	$\ln(N_1)$	p	$\ln(N_1) - \overline{\ln(N_1)}$	^{2}
1.5	0.4054651081	0.6	0.439444915	0.193111
0.5	−0.69314718	0.4	−.659167373	0.4434501

$E(\ln(N_1)) = -0.0339798$

So $Var(\ln(N_1)) = 0.289667750595$