

Corrections to problem set 1 solutions:

1.7 54 was intended to be 56. Here is the solution to 54, which is silly: In 43 years it decreases my half, and in 86 years by half again, so it is $1/4$ of the original value or 400.

1.7 58 was intended to be 60. Here is the solution to 58: $Q(t) = 6.0 \times 10^{10} e^{-0.000122t}$. We graph t vs. $\ln(Q) = \ln(6 \times 10^{10}) - 0.000122t$ over $[0, 20000] \times [0, 25]$. We get a linear function decreasing. It is at 3.0×10^{10} at $t \simeq 5700$. Notice that this is not half the height on your graph.

1.9 34 Correct problem but used $q = 0.2$ instead. For $q = 0.1$, $c_{t+1} = 0.9(c_t + S) + 0.1(0.0004)$ Solve to find $c^* = 9S + 0.0004$. If $c^* = 0.04$, then we can solve to get $S = 0.0044$

1.9 46 And I think I was just wrong on this one. Let's try doing this step by step. My writing the work makes it more likely that I am correct, and easier for you to tell me if I'm not. All volume units are m^3 . Concentrations don't have units. Suppose we start at a concentration c_t in the lake. The volume of salt before is then $3.3 \times 10^7 c_t$. The water after inflow is 3.6×10^7 . 3×10^6 water of salt concentration 0.001 flows in, so we multiply to find that $3000 = 3 \times 10^3$ salt is added, leaving a total of $3.3 \times 10^7 c_t + 3 \times 10^3$ salt after inflow. The concentration after inflow is therefore $\frac{3.3 \times 10^7 c_t + 3 \times 10^3}{3.6 \times 10^7}$. The amount of salt that flows out is the volume of water that flows out times this concentration, i.e. $1.5 \times 10^6 \left(\frac{3.3 \times 10^7 c_t + 3 \times 10^3}{3.6 \times 10^7} \right)$. So the amount of salt remaining is the amount after inflow with the outflow subtracted: $3.3 \times 10^7 c_t + 3 \times 10^3 - 1.5 \times 10^6 \left(\frac{3.3 \times 10^7 c_t + 3 \times 10^3}{3.6 \times 10^7} \right)$, and the concentration at the end is this divided by the water at the end, which is back where it started, therefore $c_{t+1} = \frac{3.3 \times 10^7 c_t + 3 \times 10^3 - 1.5 \times 10^6 \left(\frac{3.3 \times 10^7 c_t + 3 \times 10^3}{3.6 \times 10^7} \right)}{3.3 \times 10^7}$. That's a lot of numbers, but not a lot of algebra. It simplifies to $c_{t+1} = \frac{23}{24} c_t + 0.000087$, which after all this work is quite simple to solve for an equilibrium of 0.00209, or over twice the inflow concentration because of the evaporation.

1.10.45 Ok, on this one there's a typo in the book. Adler must mean $2e^{-\frac{b_t}{1.0 \times 10^6}}$, but that's neither of our faults. So, using what is in the book $b_{t+1} = (2e - \frac{b_t}{10^6})b_t$ graph $r(t)$ on $[0, 6 \times 10^6] \times [0, 6]$ and the updating function over $[0, 6 \times 10^6] \times [0, 6 \times 10^6]$. In this setting the equilibrium is at $b^* = 4.4 \times 10^6$ and is unstably oscillating, and the one at zero unstably increases. The derivative of the updating function is $2e - \frac{2b_t}{10^6}$. At 0 this is $2e > 1$, and at $b^* = 4.4 \times 10^6$ this is $2e - 8.8 < -1$.